QUALIFYING EXAMINATION

JANUARY 2002

MATH 519 - Prof. Davis

20 points/problem

1. Let $N_1, N_2, \ldots, N_{100}$ be iid (independent and identically distributed) normal mean 0 and variance 1 variables. Let $X_1, X_2, \ldots, X_{100}$ be iid variables with distribution $P(X_i = -1) = P(X_i = +1) = \frac{1}{2}$, and let the X_i also be independent of the normal variables. Find $P(\sum_{i-1}^{100} X_i N_i > 5)$. Leave your answer as an integral.

2. The arrival times $\tau_1 < \tau_2 < \cdots$ of a Poisson process (with rate $\lambda = 1$) are rounded down to the nearest tenth. Let $s_1 \leq s_2 \leq \cdots$ be the numbers this rounding produces. Put $N = \inf\{i : s_i \text{ is an integer}\}$. (So for example if $\tau_1 = 1.86$ and $\tau_2 = 3.09$, then $s_1 = 1.8$, $s_2 = 3$, $N = 2$, and $s_N = 3$.)
   (a) Find $P(s_N = 7)$.
   (b) Find $E\tau_N$.

3. Let $\mathbf{X} = (X_1, X_2)$ be uniform in the unit square $\{0 \leq x \leq 1, 0 \leq y \leq 1\}$ and let $\mathbf{Z} = (Z_1, Z_2)$ be independent of $\mathbf{X}$ and have the same distribution. Give the joint density of $\mathbf{X} + \mathbf{Z}$.

4. A fair coin is tossed 100 times and then tossed again as many times as tails were obtained in the first 100 tosses. Let X be the total heads tossed. Find the mean and variance of X in as simplified a form as possible (decimal form is best).

5. (a) Find the density of the median of three independent uniform $(0, 1)$ variables.
   (b) Is there a density g such that if X_1, X_2, X_3 are independent and have density g then the median of the X_i has a uniform $(0, 1)$ distribution?