# QUALIFYING EXAMINATION
# JANUARY 2003
# MATH 519 - Prof. Sellke

Grading: Ten points for each problem

1. Find the greatest possible value of $E(XY)$, if $X$ is Exponential ($\lambda = 1$) and $Y$ is discrete uniform on $\{1, 2\}$. Justify your answer.

2. Let $X_1, X_2, \ldots$ be iid Exponential ($\lambda = 1$) random variables. Let $M_n = \max\{X_1, \ldots, X_n\}$. Find constants $c_n$ so that the difference $D_n = M_n - c_n$ converges in distribution, and find the limiting distribution.

3. Let $X_1, X_2, \ldots$ be iid Exponential ($\lambda = 1$) random variables, and let $L_n = min\{X_1, \ldots, X_n\}$. Show that the quotient $\frac{L_{2n}}{L_n}$ converges in distribution, and find the c.d.f. of the limiting distribution.

4. Let $(X, Y)$ be a random point in $\mathbb{R}^2$, distributed uniformly on the interior of the unit circle. Find the density of $T = X + Y$.

5. Each day, starting on January 1, 2002, Professor Sellke has recorded the value of a standard normal random variable, generated according to the precepts of Professor H. Rubin. Each day's random variable is independent of those that came before. So, on December 31, 2002, Sellke recorded the 365th value on his list, and on January 1, 2003, Sellke recorded the 366th value.

   Whenever Sellke records a value bigger than all his previous values, he puts a big red R beside it, the R standing for "record". So, the value for January 1, 2003 got an R if and only if it was bigger than all the 365 values generated in 2002.

   (a) (3 points) What is the expected number of records among the 365 values that Sellke generates in the year 2003?

   (b) (7 points) Give a number which approximates the probability that there will be exactly 3 records among the 365 values for the year 2003?

6. Let $X_1, Y_1, X_2$, and $Y_2$ be independent, standard normal random variables. Let $C_1$ be the circle in $\mathbb{R}^2$ with center at the origin and the point $(X_1, Y_1)$ on its edge. Let $C_2$ be the circle in $\mathbb{R}^2$ with center at the origin and point $(X_2, Y_2)$ on its edge. Find the probability that $|\text{area}(C_1) - \text{area}(C_2)| < 1$, that is, that the area within circle $C_1$ differs from the area within circle $C_2$ by less than 1.

⋆