

QUALIFYING EXAMINATION

AUGUST 2005

MATH 519 - Prof. Davis

Questions 1, 2, 3, 4 are worth 15 points each.

Questions 5 and 6 are worth 20 points each.

1. Steve and Kyle, with 5 other men, and 5 other women, are lined up at random for taking a picture. Let X = number of men between Steve and Kyle in the lineup. Compute, exactly, $P(X = 2)$.
2. Suppose the number of children in a particular family is 1, 2, 3 or 4, with probabilities .1, .3, .4, .2 respectively. Nells, a child in this family, has no sisters. What is the probability he is the only child?
3. A fair die is repeatedly rolled until the sum of the rolls exceeds 500 for the first time. Find approximately the probability that at most 140 tosses will be necessary. You may leave your answer in the form of an integral.
4. Three lightbulbs have independent exponential parameter one lifetimes. Find the probability that none of the three failure times are within one of each other.
5. A needle of length $L < 1$ is dropped randomly onto a sheet of paper with parallel lines drawn at unit distance apart. Find explicitly the probability that the needle will cut one of the parallel lines.
6. There are N molecules of benzine mixed in with a quart of water in a jar. Half the mixture is removed and replaced with half a quart of pure water. The liquid in the jar is mixed and again half of it is removed and replaced with water, and so on. Show that there is a positive number c not depending on N such that for every N there is a probability of at least c that at some time during this process exactly one molecule of benzine remains in the jar.