QUALIFYING EXAMINATION

JANUARY 2005

MATH 519 - Prof. B. Davis

1 Let X and Y be independent exponential random variables with parameter λ. Let m and M be the smallest and largest of X and Y respectively. Find the joint density of m and $M - m$.

2 Two fair six sided dice are rolled together until the numbers on both dice are the same. Let X be the number of rolls on which the dice total seven and Y be the number of rolls on which the dice total five. Give the distribution of X, the distribution of $X+Y$, and the conditional distribution of X given $X + Y = 10$. Identify these distributions by name if you can.

3 A coin with probability p of heads is tossed one hundred times. Let X be the number of times that a toss was heads and was preceeded by a heads. For example, if the first two tosses are heads and the fifth toss is heads and all other tosses are tails, then $X = 1$, the 1 coming from the second toss being a heads preceeded by a heads. Find the mean and variance of X.
Remark: indicator random variables can be helpful.

4 A point is picked at random from the unit disc. Let X be the distance of the point from the center of the disc, Y be the distance of the point from the edge of the disc, and Z be the quadrant the point is in, that is, $Z = 1$ if both X, Y are bigger than 0, $Z = 2$ if $Y > 0$, $X < 0$, $Z = 3$ if $X, Y < 0$, and $Z = 4$ if $X > 0$, $Y < 0$. Find EX, EXY, and $EXYZ$.

5 Let X_k be continuous uniform on $(0, k)$, and let X_i, $i \geq 1$ be independent. Find $P(X_n < X_{n-1} < \cdots < X_2 < X_1)$.