

QUALIFYING EXAMINATION

JANUARY 2006

MATH 519 - Prof. Dasgupta

NOTE : 1. ANSWER TWO QUESTIONS FROM EACH GROUP. IF YOU ANSWER MORE THAN TWO QUESTIONS, ONLY THE FIRST TWO YOU HAVE ANSWERED WOULD BE GRADED. THE OTHERS WILL NOT GET GRADED.

2. STANDARD NORMAL TABLE MAY BE REQUIRED FOR SOME PROBLEMS.

Group A

1. The birthdays of 5 people are known to fall in exactly 3 calendar months. What is the probability that exactly two of the 5 were born in January ?

2. Coupons are drawn, independently, with replacement, one at a time, from a set of 10 coupons. Find, explicitly, the expected number of draws

a) until the first drawn coupon is drawn again;

b) until a duplicate occurs.

3. Let N be a positive integer. Choose an integer at random from $\{1, \dots, N\}$. Let E be the event that your chosen random number is divisible by 3, and divisible by at least one of 4 and 6, but not divisible by 5. Find, explicitly, $\lim_{N \rightarrow \infty} P(E)$.

4. Anirban is driving his Dodge on a highway with 4 lanes each way. He is wired to change lanes every minute on the minute. He changes with equal probability to either adjacent lane if there are two adjacent lanes, and the successive changes are mutually independent. Find, explicitly, the probability that after 4 minutes, Anirban is back to the lane he started from,

a) if he started at an outside lane;

b) if he started at an inside lane.

Group B

5. Burgess is going to Moose Pass, Alaska. He is driving his Dodge. He puts his car on cruise control at 70 miles per hour. Gas stations are located every 30 miles, starting from his home. His car runs out of gas at a time distributed as an Exponential with mean 4 hours. When that happens, he gets out, takes his bike out of his trunk, and bikes to the NEXT gas station, say M , at 10 miles per hour. Let the time elapsed between when Burgess starts his trip and when he arrives at the gas station M be T . Find $E(T)$.

6. A fair coin is tossed n times. Suppose X heads are obtained. Given $X = x$, let Y be generated according to the Poisson distribution with mean x . Find the unconditional variance of Y , and then find the limit of the probability $P(|Y - \frac{n}{2}| > n^{\frac{3}{4}})$, as $n \rightarrow \infty$.

7. Anirban plays a game repeatedly. On each play he wins an amount uniformly distributed in $(0, 1)$ (dollar), and then he tips the lady in charge of the game the square of the amount he has won. Then he plays again, tips again, and so on. Approximately calculate the probability that if he plays and tips six hundred times, his total winnings minus his total tips will exceed 105 dollars.

Group C

8. Anirban's dog got mad at him and broke his walking cane, first uniformly into two pieces, and then the longer piece again uniformly into two pieces. Find the probability that Anirban can make a triangle out of the three pieces of his cane.

9. Suppose X, Y, Z are iid $Exp(1)$. Find the joint density of (X, XY, XYZ) .

10. Let X be the number of Kings and Y the number of Hearts in a Bridge hand. Find the correlation between X and Y .