

Math 519
Qualifying Examination
August, 2007 – Profs. Davis and Sellke

(You may leave answers in terms of the distribution function of the standard normal distribution. Also you do not need to numerically simplify your answers. Each problem is worth twenty points.)

1. Half of the many stones on a beach are perfect cubes of edge length one, and half are perfect cubes of edge length two. Out of the vast number of stones, Tom picks one hundred at random and Mary also picks one hundred at random. Estimate the probability that the total volume of the stones Tom chose exceeds 450, and estimate the probability that the total volume of the stones Tom chose exceeds the total volume of the stones that Mary chose by at least 100.

2. Let X and Y be independent exponential random variables each with parameter one. Find the density of $|X - Y|$ and the joint density of X and $|X - Y|$.

3. A balanced six sided die is rolled ten times. Add the total of all the numbers which were rolled immediately preceding those times a six is rolled. Call this total the score. (So if the first five rolls are two and the last five are six the score is 26.) Find the mean and variance of the score.

4. Eight hands of five cards each are dealt from a shuffled deck of cards without replacement, so that there are twelve undealt cards. Find the probability that at least one of these hands has no diamonds.

5. Let $X, Y,$ and Z be iid uniform $(0,1)$ random variables. Let \min be the smallest of $X, Y, Z,$ Max be the largest, and Med be the median. Find

$$P(\text{Med} - \min < Y - X).$$