MATH 519
Qualifying Examination
August 2011 – Prof. Sellke

1. **(10 points)** Suppose X is $N(0,1)$ (i.e., standard normal) and $Y \mid X = x$ is $N(x+1,1)$.

   (a) **(3 points)** Find the marginal distribution of Y.

   (b) **(4 points)** Find the correlation between X and Y.

   (c) **(3 points)** Find $\mathbf{E}[X \mid Y = y]$.

2. **(10 points)** Let X and Y be independent standard exponential rv's, with density $f(t) = e^{-t}, t \geq 0$. Let $U = \exp(X^2 - Y)$ and $V = \exp(Y^2 - X)$. Find the value at $(1,1)$ of the continuous joint density of U and V.

3. **(10 points)** A one dimensional nonhomogeneous Poisson process has the intensity (rate) function $\lambda(t) = ct$, where c is a positive constant. Find the density of the nth arrival time for a general integer $n \geq 1$.

4. **(10 points)** A man has had much too much to drink, but is still strong enough to walk and to see where he is trying to go. He starts at the origin in $\mathbf{R}^2$. He takes iid steps, each step equally likely to be "up", "down". or "right", always of length 1: in other words, the three steps $(0,1)$, $(0,-1)$, and $(1,0)$ are of probability 1/3 each. At the first time that his horizontal position equals 100, what is the approximate numerical probability that his vertical position is greater than or equal to 10?

5. **(10 points)** Let $X_1, \ldots, X_5$ and $Y_1, \ldots, Y_5$ be independent $N(0,1)$ (i.e., standard normal) random variables. Consider the 5 points in $\mathbf{R}^2$ with coordinates $(X_k, Y_k), 1 \leq k \leq 5$. Let $D_1 < D_2 < \cdots < D_5$ be the *ordered* distances of those 5 points to the origin. Find the joint density of D_1 and D_2.