MA 519 Qualifying Exam

Name:________________________________

a) Legibly print your name above.

b) Do not open this test booklet until you are directed to do so.

c) You will have 120 min. to complete the exam. Budget your time wisely!

d) This test is closed book and closed notes. You may use a non-graphing calculator during this test.

e) Throughout the test, **show your work so that your reasoning is clear**. Otherwise no credit will be given.

f) If you need extra room, use the back of the pages. Just make sure I can follow your work.

1. Suppose that Z is a standard normal random variable and that $X = \frac{1}{1+Z^2}$. Compute the p.d.f. of the random variable X.

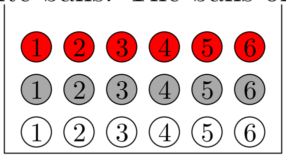
2. Suppose that A, B, and C are events with $P(A), P(B), P(C) \geq 3/4$.

a) Prove that $P(A \cap B) \geq 1/2$ and show that the lower bound cannot be improved by giving an example of events A and B where equality is achieved.

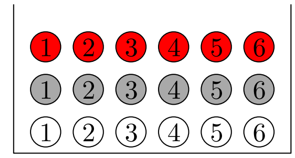
b) What is the optimal lower bound $P(A \cap B \cap C) \geq c$ that can be proved without any more assumptions on the events A, B and C? Justify your answer.

(This page intentionally left blank)

3. Suppose that a box contains 18 balls in three colors: six red balls, six black balls, and six white balls. The balls of each color are numbered 1-6.



a) If you draw 5 balls from the box without replacement, what is the probability that you have at least one ball of each color?



(continued from previous problem)

b) Suppose you draw 3 balls from the box without replacement. Let X be the sum of the numbers on the three balls drawn. Compute $E[X]$ and $\mathrm{Var}(X)$.

(This page intentionally left blank)

4. Suppose that $Z_1, Z_2, Z_3, \ldots, Z_n$ are i.i.d. Unif(0,1) random variables, and let X and Y be the largest and second largest values, respectively, in $\{Z_1, Z_2, \ldots, Z_n\}$. That is,

$$X = \max\{Z_1, Z_2, \ldots, Z_n\} \qquad Y = \max\left(\{Z_1, Z_2, \ldots, Z_n\} \backslash \{X\}\right).$$

a) Compute the joint p.d.f. of (X, Y).

(Continued from previous page)

b) Compute the marginal p.d.f. of Y.

c) Compute $E[Y]$ and $\mathrm{Var}(Y)$.

(This page intentionally left blank)

5. Let $X_1, X_2, \ldots$ be i.i.d. Unif(0,1) random variables. Compute the limit

$$\lim_{n\to\infty} P\left(\sum_{i=1}^{n} X_i \le 2\sum_{i=1}^{n} X_i^2\right).$$

Justify your answer.