Math 523 Qualifying Exam Name ____________________

1. Find the solution to the Cauchy problem

$$x\,z_x + y\,z_y = z$$

with $z = 1$ on the parabola $y = x^2 \quad (y > 0)$.

2. Describe the characteristic curves for

$$(\sin y)^2 u_{xx} + 2u_{yy} - x^2 u_y + 2u + xy = 0.$$

Explain the importance of characteristic surfaces in the study of partial differential equations.

3. State and prove a maximum principle for solutions $u(x,t)$ to the one dimensional heat equation for $0 \leq x \leq L, \quad 0 \leq t < T$.

4. State carefully the Neumann problem for Laplace's equation in a bounded domain Ω in $\mathbb{R}^3$ with smooth boundary $\partial\Omega$.

State and prove the necessary condition for the existence of a solution to the Neumannn problem for Laplace's equation.

5. Solve by Fourier series the following one dimensional wave equation with initial–boundary value conditions:

$$\begin{aligned} &u_{tt} - u_{xx} = 0 \qquad 0 < x < \pi, \quad t > 0, \\ &u(0,t) = u(\pi,t) = 0 \qquad t > 0, \\ &u(x,0) = \sin x \qquad 0 \le x \le \pi, \\ &u_t(x,0) = \sin 2x \qquad 0 \le x \le \pi \end{aligned}$$

Compute the value of the energy integral $(1/2)\int_0^\pi (u_x^2 + u_t^2)dx$ when $t = 100$.

6. Find the solution by Fourier series to the Dirichlet problem in the annulus $A = \{(r, \theta) : 1/2 < r < 1\}$,

$$\Delta u = 0$$

$$u(1/2, \theta) = \sin\theta \qquad -\pi \leq \theta \leq \pi,$$

$$u(1, \theta) = \cos 2\theta \qquad -\pi \leq \theta \leq \pi.$$