MA 530 QUALIFYING EXAMINATION

August 1996

Name: ____________________________________

1. Classify the singularities at 0:

$$a)\ \exp\left(\frac{\sin z}{z}\right), \quad b)\ \sum_{n=0}^{\infty} n(z-1)^n, \quad \cos\left(\frac{1}{e^z-1}\right).$$

2. Evaluate the integrals

$$a)\ \int_C \sin\frac{1}{z}dz \qquad b)\ \int_C \sin^2\frac{1}{z}dz,$$

where C is the circle $|z| = 2$.

3. Describe the full preimage of the segment $[-2, 2]$ under $\cos z$. Make a picture.

4. Find a conformal map of the upper half-plane, from which the vertical ray $[i, \infty)$ is removed, onto the upper half-plane.

5. Let f be a meromorphic function in the unit disc D having only one simple pole at $z_0 \in D,\ z_0 \neq 0$. Let $f(z) = a_0 + a_1 z + a_2 z^2 + ...$ in a neighborhood of 0. Prove the equality

$$z_0 = \lim_{n\to\infty} \frac{a_n}{a_{n+1}}.$$

6. Let f be a holomorphic function in the unit disc D.

   a) Prove that if f is unjective in D then $f'(z) \neq 0$ for all $z \in D$.

   b) Show that the converse is not true: there is a holomorphic function f in D whose derivative has no zeros in D but f is not injective in D.