MA 530 QUALIFYING EXAMINATION

January 1996

Name: ____________________________________

1. Find all singular points of the following functions and classify them:

   a) $\cot z - \dfrac{1}{z}$ b) $\sin\left(\exp\dfrac{1}{z}\right)$ c) $\dfrac{1}{z^2-1}\ \cos\dfrac{\pi z}{z+1}$

2. Find the Laurent expansion of

$$\frac{1}{(z-1)^2(z+2)}$$

   in the annulus $1 < |z| < 2$.

3. Evaluate the residue

$$\mathrm{Res}_{\infty} \ln\frac{z-1}{z+1}$$

   for each branch of this function which is defined in a neighborhood of ∞.

4. Find a conformal map of the following region onto the upper half–plane:

   (horizontal strip of width 2π, symmetric with respect to $\mathbb{R}$, from which the positive ray is removed).

5. For all real t evaluate the integral

$$\int_{1-i\infty}^{1+i\infty} \frac{e^{tz}}{z^2+1}\, dz$$

   (the path of integration is the vertical line $\{z : \mathrm{Re}\ z = 1\}$).

6. Show that the series

$$\sum_{n=0}^{\infty} \frac{\cos nz}{n!}$$

   is uniformly convergent on every compact in $\mathbb{C}$.