

# QUALIFYING EXAMINATION

JANUARY 1999

MATH 530 - Bell/Catlin

Notation:  $D_r(a)$  denotes the disk,  $\{z \in \mathbb{C} : |z - a| < r\}$ .

1. (15 pts) Evaluate the integral

$$\int_0^{\infty} \frac{\cos 8x}{x^2 + 1} dx.$$

2. (15 pts) Find all entire functions  $f$  such that  $f((1+i)z) = f(z)$  for all  $z \in \mathbb{C}$ .
3. (15 pts) Let  $H$  denote the half plane  $\{x + iy : y > \sqrt{2}/2\}$ . Explain how to construct an explicit one-to-one analytic map from

$$D_1(0) \cap H$$

onto  $D_1(0)$ .

4. (15 pts) Suppose that  $f$  is a non-constant entire function and suppose that  $g$  is an analytic function on  $D_1(0) \setminus \{0\}$  that has an essential singularity at 0. Prove that  $f \circ g$  must have an essential singularity at 0.
5. (20 pts) Suppose that  $u$  is a non-constant real-valued harmonic function on the whole complex plane. Show that the level set  $\{z \in \mathbb{C} : u(z) = 0\}$  must be an unbounded set.
6. (20 pts) Find all analytic functions  $f$  on  $D_2(-1) \cup D_2(1)$  such that

$$\begin{aligned} f(D_2(-1)) &\subset D_2(-1), \\ f(D_2(1)) &\subset D_2(1), \\ f(-1) &= -1 \text{ and } f(1) = 1. \end{aligned}$$