**QUALIFYING EXAMINATION**

JANUARY 2002

MATH 530 - Prof. Catlin

(15 pts) 1. Let $\Omega = \{z \in \mathbb{C};\ |z| > 1,\ \text{Re } z > 0,\ \text{Im } z > 0\}$. Find an explicit conformal map $f$ of $\Omega$ onto the unit disk. You may represent $f$ as a finite composition of maps.

(15 pts) 2. Prove that the radius of convergence of the power series $\sum_{n=0}^{\infty} a_n z^n$ is

$$R = \left(\lim_{n\to\infty} \sup |a_n|^{\frac{1}{n}}\right)^{-1}.$$

(20 pts) 3. Evaluate $\int_0^{\infty} \frac{\log x}{(x^2+1)^2}\, dx$.

(15 pts) 4. Find the number of zeros of $f(z) = 2 - z^3 - e^{-z}$ in $H = \{z;\ \text{Re } z > 0\}$.

(20 pts) 5. Find all holomorphic functions $f$ from the unit disk $D$ to itself such that $\lim_{|z|\to 1} |f(z)| = 1$.

(15 pts) 6. Let $u(z)$ be a real-valued harmonic function on the complex plane such that $K = \{z \in \mathbb{C};\ u(z) = 0\}$ is compact. Show that $u$ is identically constant.