1. Let $f$ be an unbounded entire function and $\Omega \subset \mathbb{C}$ a nonempty open set. Show that there exists $p \in \mathbb{C}$ such that $f(p) \in \Omega$.

2. Let $u: \mathbb{C} \to \mathbb{R}$ be a harmonic function. Prove that $u$ is either surjective or constant.

3. Find all entire functions $f$ such that $|f(z)| \leq |z|$ for all $z$ and $f(i) = 1$.

4. Evaluate

$$\int_{\gamma} f(z) \, dz,$$

where $f(z) = \tan((1+i)z)$, and $\gamma$ is the circle $|z| = 2$, oriented clockwise.

5. Let $\Omega = \mathbb{C} \setminus \{z \in \mathbb{R}: z \leq 0\}$. Find a bijective holomorphic function $f: \Omega \to \Omega$ such that $f(1) = i$.

6. Let $f(z) = z^{1000} + z^{100} + z^{10} + 1$. Find an $R > 0$ such that if $f(z) = 0$ then $R < |z| < R + 1$.

7. Let $\Omega \subset \mathbb{C}$ be a nonempty open set, and let $f: \Omega \to \mathbb{C}$ be holomorphic. Suppose that for every $z \in \Omega$ there is a positive integer $n$ such that $f^{(n)}(z) = 0$. Prove that $f$ is a polynomial.