QUALIFYING EXAM COVER SHEET

August 2017 Qualifying Exams

Instructions: These exams will be "blind-graded" so under the student ID number please use your PUID

ID #: ____________________________
(10 digit PUID)

EXAM (circle one) 519 523 530 **544** 553 554 562 571

For grader use:

Points _________ / Max Possible_________ Grade _________

QUALIFYING EXAM FOR MATH 544

Total: 40 points.

1. (10 pts) Let f be a measurable function on $\mathbf{R}^d$ that is finite valued a.e. Given a sequence $\{f_n\}_{n=1}^{\infty}$, with each f_n being a measurable function on $\mathbf{R}^d$ that is finite valued a.e., we say that $\underline{\{f_n\}_{n=1}^{\infty} \text{ converges to } f \text{ in measure}}$, if the following conidtion is satisfied: for any $\epsilon > 0$, we have:

$$\lim_{n\to\infty} m\big(\{x \in \mathbf{R}^d, \text{ such that } |f(x) - f_n(x)| \geqslant \epsilon\}\big) = 0$$

Now show the following. Suppose that $f \in L^1(\mathbf{R}^d)$, and $\{f_n\}_{n=1}^{\infty}$ a sequence of functions in $L^1(\mathbf{R}^d)$ such that $\{f_n\}_{n=1}^{\infty}$ converges to f in the L^1-norm. Then $\{f_n\}_{n=1}^{\infty}$ converges to f in measure.

2. (8 pts) Let $\{E_n\}_{n=1}^{\infty}$ be a sequence of measurable sets in $\mathbf{R}^d$, such that:

$$\sum_{n=1}^{\infty} m(E_n) < \infty$$

Show that $m(A) = 0$, where the set A is defined as:

$$A = \{x \in \mathbf{R}^d, \text{ such that } x \in E_n \text{ for infinitely many } n\}$$

Hint: You may use

$$A = \bigcap_{k=1}^{\infty} \bigcup_{n\geqslant k}^{\infty} E_n$$

3. (14 pts) Prove the Riesz-Fischer theorem for $L^1(\mathbf{R}^d)$: that $L^1(\mathbf{R}^d)$ is complete with respect to the L^1-norm.

4. (8 pts) Given an example of a sequence of functions $\{f_n\}_{n=1}^{\infty}$ in $L^1(\mathbf{R})$, such that

$$\lim_{n\to\infty} f_n(x) \text{ exists a.e.}$$

and that $f := \lim_{n\to\infty} f_n$ is again in $L^1(\mathbf{R})$, but

$$\lim_{n\to\infty} \int_{\mathbf{R}} f_n \neq \int_{\mathbf{R}} f$$