1. Calculate
\[ \int_{0}^{\infty} \frac{1}{x^n + 1} \, dx \]
for positive integers \( n \geq 2 \) by integrating a complex function around the closed contour that follows the real axis from the origin to \( R > 0 \), then follows the circular arc \( Re^{i\theta} \) as \( \theta \) ranges from zero to \( 2\pi/n \), then returns to the origin via the line segment joining \( Re^{2\pi i/n} \) to the origin; let \( R \to \infty \). Show all your calculations and explain all limits.

2. Describe the image of the half-strip \( \{z = x + iy : -1 < x < 1, \ 0 < y < \infty\} \) under the mapping \( f(z) = \frac{z - 1}{z + 1} \).

3. (a) Prove that \( f(z) = 1/z \) does not have a complex antiderivative in \( \mathbb{C} - \{0\} \).
   (b) Find all integers \( n = 0, \pm 1, \pm 2, \ldots \) such that the function \( g(z) = z^n e^{1/z} \) has a complex antiderivative in \( \mathbb{C} - \{0\} \).

4. Let \( f \) be an analytic function with a zero of order 2 at \( z_0 \). Prove that there exist \( \epsilon > 0 \) and \( \delta > 0 \) such that for every \( w \) in \( D_\epsilon(0) - \{0\} \), the equation \( f(z) = w \) has exactly 2 distinct roots in the set \( z \in D_\delta(z_0) - \{z_0\} \).

5. Prove that there is no analytic function that maps the punctured disc \( \{z \in \mathbb{C} : 0 < |z| < 1\} \) one-to-one onto the annulus \( \{z \in \mathbb{C} : 1 < |z| < 2\} \).