NAME: ______________________________

T.T.Moh **Math 554 Qualifying Examination** **Aug. 2013**

- This is a two hour test.
- Write your answers on the test paper!
- For decimal approximations, it is enough to give 2 decimal places.
- Show your work such that your reasoning can be followed.
- There are 10 pages, 10 questions, 20 points each and 200 points on this test.

1. Let $A \in M_{n\times n}(S)$ (the module of $n\times n$ matrices over the ring S). Prove that A is invertible iff $det(A)$ is an unit in S.

2. Let $A \in M_{3\times 3}(R)$ (the vector space of 3×3 matrices over the real field R). Show that if A is not similar over R to a trianglar matrix, then A is similar over the complex number field C to a diagonal matrix.

3. Let $M = (f_1, f_2, f_3)^T$ be a matrix over $R[x]$ where $R[x]$ is the ring of real polynomials and $f_1 = (x-3, 1, 0)$, $f_2 = (1, x-3, 0)$, $f_3 = (0, 0, x-4)$ be the three row vectors of M. Show that M is equavalent to a diagonal matrix with diagonals (c_i) for $i = 1, 2, 3$ and $c_i | c_{i+1}$.

4. Show that the product of two self-adjoint operators is self-adjoint if and only if the two operators commute.

5. Let R be the field of real numbers. Let W be the subspace of R^4 generated by $(1,0,0,0)^T, (0,0,1,1)^T$. Given $x = (1,2,1,2)^T$. Find $y, z \in R^4$ such that $x = y + z$ and $y \in W, z \in W^{\perp}$.

6. Let A be the following matrix, $A = \begin{pmatrix} 1 & 1 & 0 & 0 \\ -1 & -1 & 0 & 0 \\ -2 & -2 & 2 & 1 \\ 1 & 1 & -1 & 0 \end{pmatrix}$. Find the characteristic polynomial and the minimal polynomial of A. Can A be diagonalized over the complex numbers C?

7. Find an orthonormal basis for P_2, the vector space of all real polynomials of degree ≤ 2 under the inner product defined as

$$< f|g >= \int_0^2 fg \; dx$$

8. Let V be an inner product space (finite or infinite dimensional), show that every isometry T, i.e., $< Tv, Tu > = < v, u >$ for all $u, v \in V$, is injective.

9. Recall that an $n \times n$ matrix S over the real space R^n is said to be a rotation matrix iff S is orthogonal and $det(S) = 1$. Show that a rotation matrix A of R^3 (the real 3-dimensinal space) must have 1 as an eigenvalue.

10. Let A be the matrix over complex numbers as follows,

$$A = \begin{pmatrix} 0 & 1 & 3 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}.$$

Find matrices D, J such that $D^{-1}AD = J$ where J is the Jordan canonical form of A.