NAME: ____________________

T.T.Moh **Math 554 Qualifying Examination** **Jan. 2013**

- This is a two hour test.
- Write your answers on the test paper!
- For decimal approximations, it is enough to give 2 decimal places.
- Show your work such that your reasoning can be followed.
- There are 10 pages, 10 questions, 20 points each and 200 points on this test.

1. Let $A \in M_{n\times n}(K)$ (the vector space of $n \times n$ matrices over a field K). Show that the monic minimal polynomial of A is a factor of the characteristic polymonimal of A and all roots of the characteristic polynomial of A are roots of the minimal polynomial of A.. :

2. Let $M = (f_1, f_2, f_3)^T$ be a matrix over $R[x]$ where $R[x]$ is the ring of real polynomials and $f_1 = (x-1, 1, 0)$, $f_2 = (1, x-1, 0)$, $f_3 = (0, 0, x-2)$ be the three row vectors of M. Express M as a diagonal matrix with diagonals (c_i) for $i = 1, 2, 3$ and $c_i | c_{i+1}$.(The Smith theorem of matrices over P.I.D.)

3. Give an example of a projective module which is not free.

4. Show that $Ext^n_Z(Z/mZ, Z) = 0$ for $m, n \geq 2$.

5. Find the best straight line fit (least square approximation) to the measurement $b = 2$ *at* $t = 0$, $b = 1$ *at* $t = 1$, $b = 3$ *at* $t = 2$.

6. Find an orthonormal basis for P_2, the vector space of all real polynomials of degree ≤ 2 under the inner product defined as

$$< f|g >= \int_0^1 fg \; dx$$

7. Let V be an inner product space (finite or infinite dimensional), show that every isometry T, i.e., $< Tv, Tu >=< v, u >$ for all $u, v \in V$, is injective.

8. Show that a reflection matrix A of R^3 (the real 3-dimensinal space), i.e., a 3×3 matrix A is reflection, iff A is orthogonal and $det(A) = -1$, must have -1 as an eigenvalue.

9. Let A be the matrix over complex numbers as follows,

$$A = \begin{pmatrix} 0 & 1 & 2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}.$$

Find the Jordan canonical form of A.

10. Decide if the following function has a local minima at the origin;

$$f(x,y,z) = 2x^2 + 6xy + 2xz + 5y^2 + 3yz$$