QUALIFYING EXAM COVER SHEET

August 2017 Qualifying Exams

Instructions: These exams will be "blind-graded" so under the student ID number please use your PUID

ID #: ____________________________

(10 digit PUID)

EXAM (circle one) 519 523 530 544 553 **554** 562 571

For grader use:

Points _________ / Max Possible_________ Grade _________

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NAME: ____________________

T.T.Moh **Math 554 Qualifying Examination** **August 8th. 2017**

- This is a two hour test.
- Write your answers on the test paper!
- For decimal approximations, it is enough to give 2 decimal places.
- Show your work such that your reasoning can be followed.
- There are 10 pages, 10 questions, 20 points each and 200 points on this test.

1. Let $A \in M_{n\times n}(K)$ (the vector space of $n \times n$ matrices over a field K). Show that the monic minimal polynomial of A is a factor of the characteristic polymonimal of A and all roots of the characteristic polynomial of A are roots of the minimal polynomial of A.. :

2. Let $M = (f_1, f_2, f_3)^T$ be a matrix over $R[x]$ where $R[x]$ is the ring of real polynomials and $f_1 = (x-3, 1, 0)$, $f_2 = (1, x-3, 0)$, $f_3 = (0, 0, x-2)$ be the three row vectors of M. Express M as a diagonal matrix with diagonals (c_i) for $i = 1, 2, 3$ and $c_i | c_{i+1}$.(The Smith theorem of matrices over P.I.D.)

3. Find the area of the paralleliogram spanned by two vectors $[1, 2, 3, 4, 5]^T$ and $[5, 4, 3, 2, 1]^T$.

4. Find the singular value decomposition (SVD) of the following matrix

$$A = \begin{pmatrix} 1 & 2 \\ 1 & 1 \\ 1 & 1 \end{pmatrix}.$$

5. Find the best straight line fit (least square approximation) to the measurement $b = 2$ *at* $t = 0$, $b = 1$ *at* $t = 1$, $b = 3$ *at* $t = 2$.

6. Find an orthonormal basis for P_3, the vector space of all real polynomials of degree ≤ 3 under the inner product defined as

$$< f|g >= \int_0^1 fg \; dx$$

7. Let V be an inner product space (finite or infinite dimensional), show that every isometry T, i.e., $< Tv, Tu > = < v, u >$ for all $u, v \in V$, is injective.

8. Show that a reflection matrix A of R^3 (the real 3-dimensinal space), i.e., a 3×3 matrix A is reflection, iff A is orthogonal and $det(A) = -1$, must have -1 as an eigenvalue.

9. Let A be the matrix over complex numbers as follows,

$$A = \begin{pmatrix} 0 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}.$$

Find the Jordan canonical form of A.

10. Decide type of the following function, a ellipse? a hyperbola? a parabla?

$$2x^2 + 6xy + 2y^2 + x = 0$$