Math 562: Winter 2015 Qualifying Exam (McReynolds)

PUID Number:______________________________________

Work **four out of five** of the following problems. The time limit is two hours. Please explicitly indicate which four problems you want graded as otherwise this decision will be made for you with no guarantee it will be the optimal outcome.

Problem 1. [15 points]

Let $F\colon \mathbf{R}^4 \to \mathbf{R}^2$ be given by

$$F(x,y,w,z) = (x^2+y, x^2+y^2+w^2+z^2+y).$$

(a) Prove that $(0,1)$ is a regular value for F.

(b) Prove that $F^{-1}(\{(0,1)\})$ is diffeomorphic to the 2–sphere.

Problem 2. [15 points]

Let X,Y,Z be smooth manifolds without boundary and $f\colon X \to Z$, $g\colon Y \to Z$ be smooth maps. Prove that if f and g are transverse, then

$$W = \{(x,y) \in X \times Y \ : \ f(x) = g(y)\}$$

is a smooth manifold. Compute the dimension of W in terms of the dimensions of X,Y,Z.

Problem 3. [15 points]

Let X be an oriented compact manifold with boundary. Prove that $\chi(\partial X)$ is even.

Problem 4. [15 points]

On a smooth manifold X, we say a smooth ℓ–form ω is **divisible** by a smooth 1–form α if there exists a smooth $(\ell-1)$–form β such that $\omega = \beta \wedge \alpha$. Prove that ω is divisible by α if and only if $\omega \wedge \alpha = 0$.

Problem 5. [15 points]

Let X be a compact, orientable n–manifold with boundary ∂X and let $i\colon \partial X \to X$ be the inclusion map. For $\alpha \in \Omega^k(X)$ and $\beta \in \Omega^{n-k-1}(X)$ such that $i^*(\beta) = 0$, prove that

$$\int_X d\alpha \wedge \beta = (-1)^{k+1} \int_X \alpha \wedge d\beta.$$