QUALIFYING EXAMINATION
MATH 571
AUGUST 1994

1. a) Define metric.

   b) Consider the set of all continuous real valued functions on $[0,1]$. Show $d(f,g) := \int_0^1 |f(x) - g(x)| dx$ defines a metric on X.

2. Consider the topology on the real line generated by half open intervals $[x,y)$ and $(x,y]$. Show that this is the discrete topology.

3. Show the unit interval $[0,1]$ is connected.

4. Let X be a compact Hausdorff space, let $\{C_\alpha\}$ be a family of closed subsets so that each $C_\alpha \cap C_\beta$ is in the family. Let $C = \cap C_\alpha$ and suppose that $C \subset U$ where U is an open set. Show $C_\alpha \subset U$ for some α.

5. Give an example in the plane $\mathbb{R}^2$ of a compact connected space which is not path connected and not locally connected.

6. Let X be a Hausdorff space. Let the cone of X, denoted CX, be the quotient of $X \times [0,1]$ where $(x,1)$ is identified to a point. Show CX is locally compact only if X is compact.

7. Let Y^X denote the space of continuous maps from X to Y given the compact-open topology. Suppose X and Y are locally compact Hausdorff. Show that f and g in Y^X are in the same path component if and only if they are homotopic (Hint: Use the Exponential Law).

8. Suppose X is a path connected and locally path connected Hausdorff space so that its universal covering space $\tilde{X}$ exists. Assume X is compact. Show $\tilde{X}$ is compact only if $\pi_1(X)$ is a finite group.

9. Give an example of two compact connected metric spaces which are homotopy equivalent but are not homeomorphic.

10. Give an example of a connected finite simplicial complex K so that $X(K) = -1$. Recall that the Euler characteristic

$$X(K) := \#(0 - \text{Simplexes}) - \#(1 - \text{Simplices}) + \#(2 - \text{Simplicies}) \ldots$$