Qualifying Examination

August 1999

MATH 571 - Prof. Gottlieb

It is not necessary to prove that an example is an example.

(15 pts) 1. a) Let X be a compact Hausdorff space. If $C_1 \supseteq C_2 \supseteq C_3 \supseteq \cdots$ is a sequence of closed connected subsets of X, show that the intersection of all the subsets $\cap_1^\infty C_i$ is connected.

(10 pts) b) Show that is X is not compact, then $\cap_1^\infty C_i$ need not be connected by giving an example.

(15 pts) 2. Let $f : X \to Y$ be a continuous, closed, surjective map. If X is locally connected, then so is Y.

(10 pts) 3. Prove that $f : X \to Y$ is continuous if and only if for every subset of A of X one has $f(\overline{A}) \subset \overline{f(A)}$.

(10 pts) 4. Let

$$X = \{(x, y) \in \mathbb{R}^2 \mid x^2 + y^2 < 1\}$$
$$Y = \{(x, y) \in \mathbb{R}^2 \mid x^2 + y^2 \leq 2\}$$
$$Z = \{(x, y) \in \mathbb{R}^2 \mid 1 \leq x^2 + y^2 \leq 2\}$$

be subsets of the plane $\mathbb{R}^2$. Show that each of them is not homeomorphic to the other two (i.e. specify the topological properties that distinguish them.)

(10 pts) 5. Give an example in the plane of a compact connected space which is not path connected.

6. Suppose A is a closed simply connected subspace of the plane $\mathbb{R}^2$. Let $f : A \to S^1$ be a continuous map.

(10 pts) a) Prove that there exists a continuous map $g : \mathbb{R}^2 \to S^1$ which extends f.

(10 pts) b) Give a counterexample for A not closed.

(10 pts) c) Give a counterexample for A not simply connected.

Hint: Consider the universal covering of the circle S^1.