QUALIFYING EXAMINATION

AUGUST 2000

MATH 571 - Prof. Gottlieb

1. Let $X \times Y$ be Hausdorff and Y compact. Show that the projection map $X \times Y \to X$ is a closed map.

2. Let X be a topological space that is Hausdorff, locally compact, and second countable. Show that X is metrizable.

3. Let $f : X \to Y$ be a continuous map between compact Hausdorff spaces. Show that f is a homeomorphism if and only if it is injective and surjective.

4. We define the Pseudo-circle as follows:

   i) The graph of $y = \sin(\frac{2\pi}{x})$ for $0 < x \leq 1$.

   ii) The interval $\{(0, y) | -1 \leq y \leq 1\}$.

   iii) A path from $(0, 0)$ to $(1, 0)$ which does not intersect i) or ii) except at the end points.

   Then show that the Pseudo-circle is simply connected.

5. Let I be a unit interval. Prove that any two maps from I to Y are homotopic if and only if Y is path connected.

6. Give examples of:

   a) A map which is both open and closed.

   b) A map which is neither open nor closed.

   c) A map which is open but not closed.

   d) A map which is closed but not open.