QUALIFYING EXAMINATION

JANUARY 2000

MATH 571 - Prof. Becker

1. Let X be a compact space, let

$$C_1 \supset C_2 \supset ...C_k \supset ...$$

be a sequence of closed subsets of X, and let U be an open set such that $\bigcap_{k=1}^{\infty} C_k \subset U$. Show that there is an integer k_0 such that $C_{k_0} \subseteq U$.

2. Let X be a space. Show that X is Hausdorff if and only if the diagonal $\Delta = \{(x,x) \,|\, x \in X\}$ is closed in X.

3. Let $A \subset X$ and let X/A denote the space with the quotient topology obtained from the equivalence relation whose equivalence classes are A and the single point sets $\{x\}$, such that $x \in X - A$.

(a) Show that if X is normal and A is closed in X, then X/A is normal.

(b) Let $I = [0,1]$ and $A = \{0,1\}$. Show that I/A is homeomorphic to S^1.

4. Let X be locally compact, Hausdorff, and second countable. Show that X can be represented as a countable union of compact spaces C_k, $k = 1,2,3,...$, such that for each k, $C_k \subset \text{Int}(C_{k+1})$.

5. Let $\mathcal{C}(X,Y)$ denote the space of continuous functions from X to Y with the compact–open topology. Assume that X is locally compact Hausdorff.

(a) Show that the evaluation map

$$e : X \times \mathcal{C}(X,Y) \longrightarrow Y, \qquad e(x,f) = f(x),$$

is continuous.

(b) Let $\hat{F} : Z \longrightarrow \mathcal{C}(X,Y)$ be continuous. Use the result of part (a) to show that $F : X \times Z \longrightarrow Y$, by $F(x,z) = \hat{F}(z)(x)$, is continuous.

6. Let $p : E \longrightarrow B$ be a covering space, where E is simply connected. Show that there is a bijection $\pi_1(B; b_0) \longrightarrow p^{-1}(b_0)$.