MA 571 Qualifying Exam. January 2005.

Each problem is worth 14 points and you get two points for free.

1. Let X be a topological space.

   Let $A \subset X$ be connected.

   Prove $\bar{A}$ is connected.

2. Let $X_1, X_2, \ldots$ be topological spaces.

   Suppose that $\prod_{n=1}^{\infty} X_n$ is locally connected.

   Prove that all but finitely many X_n are connected.

3. Let X be a compact Hausdorff space.

   Let $f : X \to Y$ be a continuous **closed** surjection.

   Prove that Y is Hausdorff.

4. **Definition.** If W is a space with base point w_0 and Z is a space with base point z_0, a map $f : W \to Z$ is said to be *based* if $f(w_0) = z_0$, and a homotopy $H : W \times I \to Z$ is said to be *based* if $H(w_0, t) = z_0$ for all t.

   Let X be a space with basepoint x_0 and let $u_0 = (1, 0)$ be the base point of S^1.

   Prove that there is a 1-1 correspondence between $\pi_1(X, x_0)$ and the based homotopy classes of based continuous maps $S^1 \to X$.

5. Let B be a Hausdorff space.

   Let $p : E \to B$ be a covering map.

   Prove that E is Hausdorff.

6. Let $p : E \to B$ be a covering map.

   Let $e_0 \in E$ and let $b_0 = p(e_0)$.

   Prove that $p_* : \pi_1(E, e_0) \to \pi_1(B, b_0)$ is 1-1.

7. **Prove** that every continuous map from S^2 to S^1 is homotopic to a constant map.