MA 571 Qualifying Exam. August 2008. Professor McClure.

Each problem is worth 14 points and you get two points for free.

Unless otherwise stated, you may use anything in Munkres's book—but be careful to make it clear what fact you are using.

When you use a set theoretic fact that isn't obvious, be careful to give a clear explanation.

1. Let X and Y be topological spaces and let $f : X \to Y$ be a continuous function. **Prove** that
$$f(\overline{A}) \subset \overline{f(A)}$$
for all subsets A of X.

2. Suppose that X is connected and every point of X has a path-connected open neighborhood. **Prove** that X is path-connected.

3. Let X be a topological space and let $f, g : X \to [0, 1]$ be continuous functions.

   Suppose that X is connected and f is onto.

   Prove that there must be a point $x \in X$ with $f(x) = g(x)$.

4. Let X be the two-point set $\{0, 1\}$ with the discrete topology. Let Y be a countable product of copies of X; thus an element of Y is a sequence of 0's and 1's.

   Let A be the subset of Y consisting of sequences with only a finite number of 1's. Is A closed? **Prove or disprove.**

5. Prove the Tube Lemma: given topological spaces X and Y with Y compact, a point $x_0 \in X$, and an open set N of $X \times Y$ containing $\{x_0\} \times Y$, **prove** that there is an open set W of X containing x_0 with $W \times Y \subset N$.

6. Let X and Y be topological spaces and let $f : X \to Y$ be a continuous function. Let $x_0 \in X$ and let $y_0 = f(x_0)$.

   Find an example in which f is onto but $f_* : \pi_1(X, x_0) \to \pi_1(Y, y_0)$ is not onto. **Prove** that your example really has this property. You may use any fact from Munkres.

7. **Prove** that every continuous map from S^2 to S^1 is homotopic to a constant map (hint: use covering spaces). You may use any fact from Munkres.