MA 571 Qualifying Exam. January 2014. Professor McClure.

Each problem is worth 14 points and you get two points for free.

Please be careful that your handwriting is clear and easy to read.

Unless otherwise stated, you may use anything in Munkres's book—but be careful to make it clear what fact you are using.

When you use a set theoretic fact that isn't stated in Munkres and isn't obvious, be careful to give a clear explanation.

1. Let X be a topological space, let A be a subset of X, and let U be an open subset of X. **Prove** that $U \cap \bar{A} \subset \overline{U \cap A}$.

2. Let $\sim$ be the equivalence relation on $\mathbb{R}^2$ defined by $(x, y) \sim (x', y')$ if and only if there is a nonzero t with $(x, y) = (tx', ty')$. **Prove** that the quotient space $\mathbb{R}^2/\sim$ is compact but not Hausdorff.

3. Let X and Y be topological spaces. Let $x_0 \in X$ and let C be a compact subset of Y. Let N be an open set in $X \times Y$ containing $\{x_0\} \times C$. **Prove** that there is an open set U containing x_0 and an open set V containing C such that $U \times V \subset N$.

4. Let X be a locally compact Hausdorff space and let A be a subset with the property that $A \cap K$ is closed for every compact K. **Prove** that A is closed.

5. Let X and Y be path connected and let $h : X \to Y$ be a continuous function which induces the trivial homomorphism of fundamental groups. Let $x_0, x_1 \in X$ and let f and g be paths from x_0 to x_1. **Prove** that $h \circ f$ and $h \circ g$ are path homotopic.

6. Let X be the quotient space obtained from an 8-sided polygonal region P by pasting its edges together according to the labelling scheme $aabbcdc^{-1}d^{-1}$.

   i) Calculate $H_1(X)$. (You may use any fact in Munkres, but be sure to be clear about what you're using.)

   ii) Assuming X is homeomorphic to one of the standard surfaces in the classification theorem, which surface is it? Justify your answer.

7. Let $p : E \to B$ be a covering map with B locally connected, and let $x \in B$. **Prove** that x has a neighborhood W with the following property: for every connected component C of $p^{-1}(W)$, the map $p : C \to W$ is a homeomorphism.