Math 571 Qualifying Exam **January 2018**

Each problem is worth ten points, for a total of sixty possible points.

1. Let X and Y be topological spaces and let $f, g : X \to Y$ be two continuous functions. Suppose that Y is Hausdorff. Show that the subset of X consisting of those points $x \in X$ where $f(x) = g(x)$ is closed.

2. Let X and Y be (nonempty) connected topological spaces. Show that $X \times Y$ is connected.

3. A collection of subsets $\{Z_\alpha\}_{\alpha \in A}$ of a topological space X is said to have the *finite intersection property* if every finite subcollection $\{Z_{\alpha_1}, \dots, Z_{\alpha_n}\}$ of $\{Z_\alpha\}_{\alpha \in A}$ has nonempty intersection. Show that a topological space X is compact if and only if, for each collection of *closed* subsets $\{Z_\alpha\}_{\alpha \in A}$ of X having the finite intersection property, the total intersection $\bigcap_{\alpha \in A} Z_\alpha$ is nonempty.

4. Let X be a locally compact Hausdorff space and let $Z \subset X$ be a subset with the property that $Z \cap K$ is closed for every compact $K \subset X$. Prove that Z is closed.

5. Let X be the quotient of the square $[0,1] \times [0,1]$ by the equivalence relation generated by $[s, 0] \sim [s, 1]$ and $[0, t] \sim [1, 1-t]$ for all $s, t \in [0, 1]$. Show that X is path connected and calculate $\pi_1(X)$.

6. Let $X = S^1$ be the circle and let $p : Y \to X$ be a covering space. Show that if Y is path connected but not simply connected then $\pi_1(Y) \cong \mathbb{Z}$.