Purdue University

Study Guide for MA 16020 Credit Exam

Students who pass the credit exam will gain credit in MA16020. The credit exam is a two-hour long exam with 25 multiple choice questions. No books or notes are allowed. A formula sheet will be provided and is shown later in this document. Only **one-line display** calculators may be used. No exceptions! A calculator with exponential, logarithmic and basic trigonometric functions will be needed for the exam. No any other electronic devices may be used.

To prepare for the exam, you are being provided:

1. Lesson Plan for MA16020
2. Exam Formulas that will be provided with the exam
3. Practice Problems

The lesson plan lists the sections of the text that are covered in MA16020. The practice problems provide some preparation for the exam.

The current text used for MA16020 as well as the course web page are listed below (A copy of the MA16020 text is on reserve in the math library.) Most of the material covered on the credit exam can be studied from any calculus textbook.

Textbook:	Applied Calculus and Differential Equations
Edition:	Purdue Custom First Edition
Authors:	Larson, Edwards, Zill and Wright

The url for the course web page is: http://www.math.purdue.edu/academic/courses/MA16020/

The book listed above is a custom made, loose leaf text for MA16010, MA16020 and MA16021. A big portion of the text comes from the book listed below, which contains most of the topics covered in MA16020. The following text is also on reserve in the math library. To prepare for the MA16020 credit exam, you may use either of these two texts. The section numbers listed on the lesson plan on the next page apply to both texts.

Textbook:	Calculus of a Single Variable
Edition:	Sixth Edition
Authors:	Larson and Edwards

When you are ready for the examination, obtain the proper form from your academic advisor. Follow the instructions on the form. Good luck!

MA 16020 Applied Calculus I Lesson Plan

Lesson	**Section**	**Topic**
1	5.5	Integration By Substitution
2	5.5	Integration By Substitution
3	5.7	The Natural Logarithmic Function: Integration
4	6.2	Diff. Equations: Solutions,Growth and Decay
5	6.3	Diff. Equations: Separation of Variables
6	6.3	Diff. Equations: Separation of Variables
7	6.5	First-Order Linear Differential Equations
8	6.5	First-Order Linear Differential Equations
9	7.1	Area of a Region Between two curves
10	7.2	Volume of Solids of Revolution
11	7.2	Volume of Solids of Revolution
12	8.2	Integration by Parts
13	8.2	Integration by Parts
14	8.8	Improper Integrals
15	9.2	Geometric Series and Convergence
16	9.6	The Ratio Test
17	9.7	Taylor Polynomials and Approximations
18	9.8	Power Series
19	9.9	Finding Power Series Representations
20	9.10	Taylor and Maclaurin Series
21	31.1	Functions of Several Variables Intro
22	13.3	Partial Derivatives
23	13.4	Differentials of Multivariable Functions
24	13.5	Chain Rule, Functions of Several Variables
25	13.8	Extrema of Functions of Two Variables
26	13.9	Applications of Extrema -Two Var. Functions
27	13.10	LaGrange Multipliers - Constrained Min/Max
28	13.10	LaGrange Multipliers - Constrained Min/Max
29	14.1	Iterated Integrals
30	14.2	Double Integrals, Volume, Applications
31	Lar-8.1,8.2,8.3	Systems of Equations, Matrices
32	Lar-8.3	Gaussian Elimination
33	Lar-8.3,8.4	Gauss-Jordan Elimination & Matrix Operations
34	Lar-8.4,8.5	Matrix Operations, Inverse Matrices, Determinants
35	Zill-App2	Matrix Operations, Determinants
36	Zill-App2	Eigenvalues and Eigenvectors

MA 16020 – EXAM FORMULAS

THE SECOND DERIVATIVE TEST

Suppose f is a function of two variables x and y, and that all the second-order partial derivatives are continuous. Let

$$d = f_{xx}f_{yy} - (f_{xy})^2$$

and suppose (a, b) is a critical point of f.

1. 1.If $d(a, b) > 0$ and $f_{xx}(a, b) > 0$, then f has a relative minimum at (a, b).
2. 2.If $d(a, b) > 0$ and $f_{xx}(a, b) < 0$, then f has a relative maximum at (a, b).
3. 3.If $d(a, b) < 0$, then f has a saddle point at (a, b).
4. 4.If $d(a, b) = 0$, the test is inconclusive.

LAGRANGE EQUATIONS

For the function $f(x, y)$ subject to the constraint $g(x, y) = c$, the Lagrange equations are

$$f_x = \lambda g_x \qquad f_y = \lambda g_y \qquad g(x, y) = c$$

GEOMETRIC SERIES

If $0 < |r| < 1$, then

$$\sum_{n=0}^{\infty} ar^n = \frac{a}{1-r}$$

TAYLOR SERIES

The Taylor series of $f(x)$ about $x = c$ is the power series

$$\sum_{n=0}^{\infty} a_n(x-c)^n \quad \text{where} \quad a_n = \frac{f^{(n)}(c)}{n!}$$

Examples:

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}, \text{ for } -\infty < x < \infty; \qquad \ln x = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n}(x-1)^n, \text{ for } 0 < x \le 2$$

VOLUME & SURFACE AREA

Right Circular Cylinder

$V = \pi r^2 h$

$$SA = \begin{cases} 2\pi r^2 + 2\pi r h \\ \pi r^2 + 2\pi r h \end{cases}$$

Sphere

$V = \frac{4}{3}\pi r^3$

$SA = 4\pi r^2$

Right Circular Cone

$V = \frac{1}{3}\pi r^2 h$

$SA = \pi r\sqrt{r^2 + h^2} + \pi r^2$

Practice Problems

1. Evaluate $\int \frac{1}{(3x-1)^4} dx$.

   A. $-\frac{12}{(3x-1)^5}+C$

   B. $-\frac{1}{9(3x-1)^3}+C$

   C. $\frac{1}{(3x-1)^3}+C$

   D. $-\frac{1}{3(3x-1)^3}+C$

   E. $-\frac{4}{(3x-1)^5}+C$

2. Evaluate $\int e^{3-2x} dx$.

   A. $-2e^{3-2x}+C$

   B. $-\frac{1}{2}e^{3-2x}+C$

   C. $\frac{e^{4-2x}}{4-2x}+C$

   D. $\frac{1}{3}e^{3-2x}+C$

   E. $\frac{e^{3-2x}}{3-2x}+C$

3. Find a function f whose tangent line has slope $x\sqrt{5-x^2}$ for each value of x and whose graph passes through the point (2,10).

   A. $f(x)=-\frac{1}{3}(5-x^2)^{3/2}$

   B. $f(x)=\frac{2}{3}(5-x^2)^{3/2}+\frac{28}{3}$

   C. $f(x)=\frac{1}{3}(5-x^2)^{3/2}+\frac{29}{3}$

   D. $f(x)=-\frac{1}{3}(5-x^2)^{3/2}+\frac{31}{3}$

   E. $f(x)=\frac{3}{2}(5-x^2)^{3/2}+\frac{17}{2}$

4. Evaluate $\int x\ln(x^2)dx$.

   A. $\frac{1}{2}x^2\ln x^2-\frac{1}{2}x^2+C$

   B. $\frac{1}{2}x^2\ln x^2-\frac{1}{2}x+C$

   C. $\frac{1}{2}x^2\ln x^2-\frac{1}{6}x^3+C$

   D. $x\ln x^2+\frac{1}{x}+C$

   E. $\frac{1}{2}x^2\ln x-\frac{1}{2}x^2+C$

5. The area of the region bounded by the curves $y = x^2 + 1$ and $y = 3x + 5$ is

A. $\frac{125}{6}$
B. $\frac{56}{3}$
C. $\frac{27}{2}$
D. $\frac{25}{6}$
E. $\frac{32}{3}$

6. If $f(x, y) = (xy + 1)^2 - \sqrt{y^2 - x^2}$, evaluate $f(-2, 1)$.

A. 1
B. $1 - \sqrt{5}$
C. Not defined
D. $-1 - \sqrt{5}$
E. $-1 - \sqrt{3}$

7. A paint store carries two brands of latex paint. Sales figures indicate that if the first brand is sold for x_1 dollars per gallon and the second for x_2 dollars per gallon, the demand for the first brand will be $D_1(x_1, x_2) = 100 + 5x_1 - 10x_2$ gallons per month and the demand for the second brand will be $D_2(x_1, x_2) = 200 - 10x_1 + 15x_2$ gallons per month. Express the paint store's total monthly revenue, R, as a function of x_1 and x_2.

A. $R = x_1 D_1(x_1, x_2) + x_2 D_2(x_1, x_2)$
B. $R = D_1(x_1, x_2) + D_2(x_1, x_2)$
C. $R = D_1(x_1, x_2) D_2(x_1, x_2)$
D. $R = x_2 D(x_1, x_2) + x_1 D_2(x_1, x_2)$
E. $R = x_1 x_2 + D_1(x_1, x_2) D_2(x_1, x_2)$

8. Compute $\frac{\partial z}{\partial x}$, where $z = \ln(xy)$.

A. $\frac{1}{x}$
B. $\frac{1}{y}$
C. $\frac{1}{xy}$
D. $\frac{1}{x} + \frac{1}{y}$
E. $\frac{y}{x}$

9. Compute f_{uv} if $f = uv + e^{u+2v}$.

A. 0
B. $u + 2e^{u+2v}$
C. $v + 2e^{u+2v}$
D. $1 + 2e^{u+2v}$
E. $1 + e^{u+2v}$

10. Find and classify the critical points of $f(x, y) = (x-2)^2 + 2y^3 - 6y^2 - 18y + 7$.

A. (2,3) saddle point; (2,−1) relative minimum

B. (2,3) relative maximum; (2,−1) relative minimum

C. (2,3) relative minimum; (2,−1) relative maximum

D. (2,3) relative maximum; (2,−1) saddle point

E. (2,3) relative minimum; (2,−1) saddle point

11. A manufacturer sells two brands of foot powder, brand A and brand B. When the price of A is x cents per can and the price of B is y cents per can the manufacturer sells $40-8x+5y$ thousand cans of A and $50+9x-7y$ thousand cans of B. The cost to produce A is 10 cents per can and the cost to produce B is 20 cents per can. Determine the selling price of brand A which will maximize the profit.

A. 40 cents

B. 45 cents

C. 15 cents

D. 50 cents

E. 35 cents

12. Use increments to estimate the change in z at (1,3) if $\frac{\partial z}{\partial x} = 2x - 4$, $\frac{\partial z}{\partial y} = 2y + 7$, the change in x is 0.3 and the change in y is 0.5.

A. 7.1

B. 2.9

C. 4.9

D. 5.9

E. 6.3

13. Using x worker-hours of skilled labor and y worker-hours of unskilled labor, a manufacturer can produce $f(x, y) = x^2y$ units. Currently 16 worker-hours of skilled labor and 32 worker-hours of unskilled labor are used. If the manufacturer increases the unskilled labor by 10 worker-hours, use calculus to estimate the corresponding change that the manufacturer should make in the level of skilled labor so that the total output will remain the same.

A. Reduce by 4 hours.

B. Reduce by 10 hours.

C. Reduce by $\frac{5}{4}$ hours.

D. Reduce by $\frac{5}{2}$ hours.

E. Reduce by 5 hours.

14. Find the maximum value of the function $f(x, y) = 20x^{3/2}y$ subject to the constraint $x + y = 60$. Round your answer to the nearest integer.

A. 84,654

B. 188,334

C. 4,320

D. 259,200

E. 103,680

15. Evaluate $\int_1^2 \int_0^1 (2x + y)\, dydx$.

A. $\frac{9}{2}$

B. $\frac{5}{2}$

C. $\frac{3}{2}$

D. $\frac{7}{2}$

E. $\frac{1}{2}$

16. The general solution of the differential equation $\frac{dy}{dx} = 2y + 1$ is:

A. $x = y^2 + y + C$

B. $2y + 1 = Ce^{2x}$

C. $y = 2xy + x + C$

D. $y = Ce^{2x} - 2y - 1$

E. $y = Ce^{2x}$

17. The value, V, of a certain \$1500 IRA account grows at a rate equal to 13.5% of its value. Its value after t years is:

A. $V = 1500e^{-0.135t}$

B. $V = 1500 + 0.135t$

C. $V = 1500e^{0.135t}$

D. $V = 1500(1 + 0.135t)$

E. $V = 1500 \ln(0.135t)$

18. It is estimated that t years from now the population of a certain town will be increasing at a rate of $5 + 3t^{2/3}$ hundred people per year. If the population is presently 100,000, by how many people will the population increase over the next 8 years?

A. 100

B. 9,760

C. 6,260

D. 24,760

E. 17,260

19. Calculate the improper integral $\int_0^\infty xe^{-x^2}\,dx$.

A. $-\frac{1}{2}$
B. 1
C. $\frac{1}{2}$
D. $\frac{5}{2}$
E. The integral diverges.

20. An object moves so that its velocity after t minutes is given by the formula $v = 20e^{-0.01t}$. The distance it travels during the 10th minute is

A. $\int_0^{10} 20e^{-0.01t}dt$
B. $\int_9^{10} (-20e^{-0.01t})dt$
C. $\int_0^{10} (-20e^{-0.01t})dt$
D. $\int_9^{10} 20e^{-0.01t}dt$
E. $\int_9^{10} (-0.2e^{-0.01t})dt$

21. Find the sum of the series $\sum_{n=1}^{\infty}\left(-\frac{2}{3}\right)^n$.

A. $\frac{2}{5}$
B. $-\frac{2}{5}$
C. $\frac{3}{2}$
D. $-\frac{3}{2}$
E. The series diverges.

22. Use a Taylor polynomial of degree 2 to approximate $\int_0^{0.1} \frac{100}{x^2+1}\,dx$. Round your answer to five decimal places.

A. 9.96687
B. 10.00000
C. 9.96677
D. 9.66667
E. 9.96667

23. Find the radius of convergence of the power series $\sum_{n=0}^{\infty} \frac{n3^n x^n}{5^{n+1}}$.

A. $\frac{5}{3}$
B. 1
C. $\frac{3}{25}$
D. $\frac{3}{5}$
E. ∞

24. Find the Taylor series of $f(x) = \frac{x}{2+x^2}$ at $x = 0$.

A. $\sum_{n=0}^{\infty} \frac{x^{n+1}}{2^{n+1}}$
B. $\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{2^n}$
C. $\sum_{n=0}^{\infty} (-1)^n 2^{n-1} x^{2n+1}$
D. $\sum_{n=0}^{\infty} \frac{x^{2n}}{2^{n-1}}$
E. $\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{2^{n+1}}$

25. Write the following infinite series in summation notation.

$$5 - \frac{7}{8} + \frac{9}{27} - \frac{11}{64} + \dots$$

A. $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{2n+5}{n^3}$
B. $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{2n+3}{n^3}$
C. $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{3n+2}{n^3}$
D. $\sum_{n=1}^{\infty} (-1)^{n} \frac{2n+5}{2^n}$
E. $\sum_{n=1}^{\infty} (-1)^{n} \frac{2n+3}{2^n}$

26. Determine which of the following series converge.

I. $\sum_{k=2}^{\infty} \frac{k^2}{5^k}$

II. $\sum_{k=3}^{\infty} \frac{(3k+1)\pi^{2k}}{10^{k+1}}$

III. $\sum_{k=1}^{\infty} \frac{k!}{(-2)^k}$

A. III

B. I & II

C. I & III

D. II & III

E. II

27. Find the Taylor series about $x = 0$ for the indefinite integral

$$\int xe^{-x^3}\,dx.$$

A. $\sum_{n=0}^{\infty} \frac{1}{n!(3n+1)} x^{3n+2} + C$

B. $\sum_{n=0}^{\infty} \frac{(-1)^n}{n!(3n+2)} x^{3n+2} + C$

C. $\sum_{n=0}^{\infty} \frac{(-1)^n}{n!(3n+1)} x^{3n+2} + C$

D. $\sum_{n=0}^{\infty} \frac{1}{n!(3n+2)} x^{3n+2} + C$

E. $\sum_{n=0}^{\infty} \frac{(-1)^n}{n!(3n+1)} x^{3n+1} + C$

28. A patient is given an injection of 50 milligrams of a drug every 24 hours. After t days, the fraction of the drug remaining in the patient's body is

$$f(t) = 2^{-t/3}.$$

If the treatment is continued indefinitely, approximately how many milligrams of the drug will eventually be in the patient's body just prior to an injection?

A. 202.7

B. 152.7

C. 305.4

D. 242.4

E. 192.4

29. Compute $\int (\sin x - \cos x)(\sin x + \cos x)^5 \, dx$.

A. $\frac{1}{6}(-\cos x + \sin x)^6 + C$
B. $-6(-\cos x + \sin x)^6 + C$
C. $-\frac{1}{6}(\sin x + \cos x)^6 + C$
D. $6(\sin x + \cos x)^6 + C$
E. $\frac{1}{6}(\sin x + \cos x)^6 + C$

30. Evaluate $\int x^2 \cos(-5x) \, dx$.

A. $-\frac{1}{5}x^2 \sin(-5x) + \frac{2}{25}x \cos(-5x) + \frac{2}{125} \sin(-5x) + C$
B. $\frac{1}{5}x^2 \sin(-5x) - \frac{2}{25}x \cos(-5x) - \frac{2}{125} \sin(-5x) + C$
C. $-5x^2 \sin(-5x) + 50x \cos(-5x) + 250 \sin(-5x) + C$
D. $5x^2 \cos(-5x) - 50x \sin(-5x) - 250 \cos(-5x) + C$
E. $5x^2 \sin(-5x) - 50x \cos(-5x) - 250 \sin(-5x) + C$

31. Evaluate $\int_e^5 \frac{\ln(x^4)}{x} \, dx$.

A. $\frac{1}{8}(25 - e^2)$
B. $2(25 - e^2)$
C. $2(\ln 5)^2 - 2$
D. $\frac{1}{8}(\ln 5)^2 - \frac{1}{8}$
E. $\ln(25) - 2$

32. Find the volume of the solid generated by revolving the region bounded by:

$$y = 3e^{2x}, y = 0, x = 1, \text{ and } x = 3$$

about the x-axis.

A. $\frac{3\pi}{4}(e^8 - 1)e^4$
B. $\frac{3\pi}{4}(e^8 - 1)e^2$
C. $\frac{9\pi}{2}(e^4 - 1)e^2$
D. $\frac{9\pi}{4}(e^8 - 1)e^4$
E. $\frac{3\pi}{2}(e^4 - 1)e^2$

33. Find the volume of the solid which has square cross-sections with side length $5x^2$ at each point $2 \leq x \leq 4$.

A. $\int_2^4 5\pi x^2 \, dx$

B. $\int_2^4 5x^2 \, dx$

C. $\int_2^4 5x^4 \, dx$

D. $\int_2^4 25\pi x^4 \, dx$

E. $\int_2^4 25x^4 \, dx$

34. The velocity of a car over the time period $0 \leq t \leq 3$ is given by the function

$$v(t) = 60te^{\frac{-t}{4}}$$

miles per hour, where t is time in **hours**. What was the distance the car traveled in the first 90 **minutes**? Round your answer to two decimal places.

A. 166.42 miles

B. 156.19 miles

C. 126.63 miles

D. 75.85 miles

E. 52.78 miles

35. Given that $f(x, y) = \tan(xy^3)$, compute $f_x(2\pi, \frac{1}{2})$.

A. $\frac{3}{2}$

B. $\frac{\pi}{2}$

C. 1

D. 6π

E. $\frac{1}{4}$

36. Let $h(x, y) = y\sin(xy)$. Find $\frac{\partial^2 h}{\partial y \partial x}$.

A. $-2xy\sin(xy)$

B. $2y\cos(xy) - xy^2\sin(xy)$

C. $-y^3\sin(xy)$

D. $\cos(xy) + y^2\sin(xy)$

E. $(x + 1)\cos(xy) - x^2y\sin(xy)$

37. A nature preserve wishes to construct a large compound which will hold both lions and gazelles. They currently have 6 gazelles. They estimate that if they use an area of A square miles and introduce L lions, then they will be able to support a population of G gazelles, given by the function

$$G(A, L) = 6 + 40A - A^2 - 18L^2 + 176L - 8AL$$

What conditions will lead to the largest number of gazelles?

A. $L = 3,\ A = 5$

B. $L = 4,\ A = 4$

C. $L = 5,\ A = 4$

D. $L = 5,\ A = 3$

E. There are no such conditions because the function does not have a maximum.

38. Evaluate $\iint_R (e^{x^2+1})\ dA$, where R is the region indicated by the boundaries below:

$$0 \leq x \leq 1; \quad 0 \leq y \leq x$$

A. 0

B. $\frac{1}{2}e$

C. $\frac{1}{2}e^2$

D. $\frac{1}{2}(e^2 - e)$

E. $e^2 - e$

39. Compute AB and BA, if possible, for the matrices:

$$A = \begin{bmatrix} 2 & -1 \\ 0 & -3 \end{bmatrix} \text{ and } B = \begin{bmatrix} 0 & 1 \\ -5 & 1 \\ 2 & 0 \end{bmatrix}$$

A. BA is not possible, and $AB = \begin{bmatrix} -1 & -3 \\ -11 & -3 \\ 4 & 0 \end{bmatrix}$

B. BA is not possible, and $AB = \begin{bmatrix} -1 & -11 & 4 \\ -3 & -3 & 0 \end{bmatrix}$

C. AB is not possible, and $BA = \begin{bmatrix} 0 & -3 \\ -10 & 2 \\ 4 & -2 \end{bmatrix}$

D. AB is not possible, and $BA = \begin{bmatrix} 0 & -10 & 4 \\ -3 & 2 & -2 \end{bmatrix}$

E. Both AB and BA are not possible.

40. Find the general solution to the differential equation

$$-x^5 \sin x + xy' = 3y, \qquad x > 0$$

A. $y = -x \cos x - \sin x + C$

B. $y = -x \cos x + \sin x + C$

C. $y = x \cos x + \sin x + C$

D. $y = -x^4 \cos x + x^3 \sin x + Cx^3$

E. $y = x^4 \cos x + x^3 \sin x + Cx^3$

41. The amount of carbon, in grams, in a sample of soil is given by a function, $F(t)$, satisfying the differential equation:

$$F' + aF - b = 0$$

where a and b are constants, and time, t, is measured in years. If the sample originally contains 10 grams of carbon, which expression represents the amount of carbon present after 5 years?

A. $\frac{b}{a} + (10 - \frac{b}{a})e^{5a}$

B. $\frac{b}{a} + (10 - \frac{b}{a})e^{-5a}$

C. $ab + (10 - ab)e^{-5a}$

D. $ab + (10 - ab)e^{5a}$

E. $\frac{b}{a} + 10e^{5a}$

42. Let $M = \begin{matrix} 4 & 3 \\ -2 & -1 \end{matrix}$. Compute $3M - M^2$.

A. $\begin{matrix} 2 & 0 \\ 0 & 2 \end{matrix}$

B. $\begin{matrix} 2 & 1 \\ 0 & 2 \end{matrix}$

C. $\begin{matrix} 2 & 0 \\ -1 & 2 \end{matrix}$

D. $\begin{matrix} 2 & 1 \\ 1 & 2 \end{matrix}$

E. $\begin{matrix} 2 & -1 \\ 1 & 2 \end{matrix}$

43. Write the following augmented matrix in reduced row-echelon form.

$$\left[\begin{array}{ccc|c} 2 & -3 & 2 & 1 \\ 1 & -6 & 1 & 2 \\ -1 & -3 & -1 & 1 \end{array}\right]$$

A. $\left[\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & -\frac{1}{3} \\ 0 & 0 & 0 & 0 \end{array}\right]$

B. $\left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -\frac{1}{3} \\ 0 & 0 & 1 & 0 \end{array}\right]$

C $\left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -\frac{1}{3} \\ 0 & 0 & 0 & 0 \end{array}\right]$

D $\left[\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & -\frac{1}{3} \\ 0 & 0 & 1 & 0 \end{array}\right]$

E. $\left[\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & -\frac{1}{3} \\ 0 & 1 & 0 & 0 \end{array}\right]$

44. Find all the eigenvalues of the matrix $\begin{matrix} 9 & 20 \\ -6 & -13 \end{matrix}$.

A. -5 and 2

B. -3 and -1

C. -4 and 0

D. 3 and 7

E. 2 and -2

45. Find the determinant of the matrix A, and determine if A is invertible.

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 2 & -3 & 1 \end{bmatrix}$$

A. A is not invertible because $\det(A) = 9$.

B. A is invertible because $\det(A) = 9$.

C. A is not invertible because $\det(A) = -9$.

D. A is invertible because $\det(A) = -9$.

E. A is not invertible because $\det(A) = 0$.

46. The inverse of a certain Leslie matrix

$$G = \begin{matrix} 1/2 & 2 \\ 1/2 & 1/2 \end{matrix}$$

is

$$G^{-1} = \begin{matrix} -2/3 & 8/3 \\ 2/3 & -2/3 \end{matrix} .$$

If the population vector in **year 2** is $p_2 = \begin{matrix} \text{hatchlings} \\ \text{adults} \end{matrix} = \begin{matrix} 129 \\ 72 \end{matrix}$, then the population vector in **year 1**, $p_1 = \begin{matrix} \text{hatchlings} \\ \text{adults} \end{matrix} =$

A. $\begin{matrix} -2/3 & 8/3 \\ 2/3 & -2/3 \end{matrix} \quad \begin{matrix} 129 \\ 72 \end{matrix}$

B. $\begin{matrix} 1/2 & 2 \\ 1/2 & 1/2 \end{matrix} \quad \begin{matrix} 129 \\ 72 \end{matrix}$

C. $\begin{matrix} -2/3 & 8/3 \\ 2/3 & -2/3 \end{matrix} \quad \begin{matrix} 1/2 & 2 \\ 1/2 & 1/2 \end{matrix} \quad \begin{matrix} 129 \\ 72 \end{matrix}$

D. $\begin{matrix} 1/2 & 2 \\ 1/2 & 1/2 \end{matrix} \quad \begin{matrix} -2/3 & 8/3 \\ 2/3 & -2/3 \end{matrix} \quad \begin{matrix} 129 \\ 72 \end{matrix}$

E. $\begin{matrix} -2/3 & 8/3 \\ 2/3 & -2/3 \end{matrix} + \begin{matrix} 1/2 & 2 \\ 1/2 & 1/2 \end{matrix} \quad \begin{matrix} 129 \\ 72 \end{matrix}$

47. Which of the following are eigenvectors of the matrix $\begin{matrix} 0 & 6 \\ 1 & 1 \end{matrix}$?

$$\text{I. } \begin{matrix} -3 \\ 1 \end{matrix} \qquad \text{II. } \begin{matrix} 3 \\ 3 \end{matrix} \qquad \text{III. } \begin{matrix} -2 \\ -1 \end{matrix}$$

A. I only

B. II only

C. I and II only

D. I and III only

E. II and III only

48. A 2×2 matrix A has eigenvalues $r_1 = 2$ and $r_2 = -1$. The corresponding eigenvectors are given by $v_1 = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$ and $v_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ respectively. Compute $A^4 \begin{pmatrix} -2 \\ -1 \end{pmatrix}$.

A. $\begin{pmatrix} -32 \\ 35 \end{pmatrix}$

B. $\begin{pmatrix} 24 \\ -15 \end{pmatrix}$

C. $\begin{pmatrix} -16 \\ 17 \end{pmatrix}$

D. $\begin{pmatrix} -32 \\ 29 \end{pmatrix}$

E. $\begin{pmatrix} 21 \\ -28 \end{pmatrix}$

Answers to Practice Problems

1. B	2. B	3. D	4. A
5. A	6. C	7. A	8. A
9. D	10. E	11. A	12. D
13. D	14. E	15. D	16. B
17. C	18. B	19. C	20. D
21. B	22. E	23. A	24. E
25. B	26. B	27. B	28. E
29. C	30. A	31. C	32. D
33. E	34. E	35. E	36. B
37. B	38. D	39. C	40. D
41. B	42. A	43. A	44. B
45. B	46. A	47. D	48. D