Purdue University

Study Guide for MA 16021 Credit Exam

Students who pass the credit exam will gain credit in MA16021. The credit exam is a two-hour long exam with 25 multiple choice questions. No books or notes are allowed. A formula sheets (included with this study guide) will be provided. Any brand of **one-line display** calculators may be used. A calculator with exponential, logarithmic and basic trigonometric functions will be needed for the exam. No any other electronic devices may be used.

To prepare for the exam, you are being provided:

1. Lesson Plan for MA16021
2. Practice Problems

The lesson plan lists the sections of the text that are covered in MA16021. The practice problems provide some preparation for the exam.

The current text used for MA16021 as well as the course web page are listed below (A copy of the MA16021 text is on reserve in the math library.) Most of the material covered on the credit exam can be studied from any calculus textbook.

Textbook:	Applied Calculus and Differential Equations
Edition:	Purdue Custom First Edition
Authors:	Larson, Edwards, Zill and Wright

The url for the course web page is: http://www.math.purdue.edu/academic/courses/MA16021/

The book listed above is a custom made, loose leaf text for MA16010, MA16020 and MA16021. A big portion of the text comes from the book listed below, which contains all the topics covered in MA16021. The following text is also on reserve in the math library. To prepare for the MA16021 credit exam, you may use either of these two texts. The section numbers listed on the lesson plan on the next page apply to both texts.

Textbook:	Calculus of a Single Variable
Edition:	Sixth Edition
Authors:	Larson and Edwards

When you are ready for the examination, obtain the proper form from your academic advisor. Follow the instructions on the form.

MA 16021 Lesson Plan

Lesson	Topic
1-3	Integration by substituion and integration by parts
4-5	Integration of rational functions
6-7	Volumes of solids of revolution
8	Centroids
9-12	Work and Fluid pressure
13	Improper integrals
14	Infinite series
15	Maclaurin and Taylor series, operations with series
16-17	Computations with series; applications
18-19	Fourier series
20-21	First-order linear differential equations; separable differential equations
22	First-order linear differential equations
23-24	Applications of first-order linear differential equations
25-26	Higher-order homogeneous differential equations
27-29	Nonhomogeneous differential equations
30-32	Introduction and basic properties of the Laplace transform
33-34	Solutions of linear differential equations by Laplace transforms

The Formula Page may be used. It will be attached to the credit exam.

1. Evaluate $\int \sqrt{2x+1}\, dx$
   (A) $\frac{2}{3}(2x+1)^{3/2}+C$ (B) $\frac{1}{3}(2x+1)^{3/2}+C$ (C) $(2x+1)^{-1/2}+C$ (D) $2(2x+1)^{1/2}+C$
   (E) None of these

2. Evaluate $\int \frac{x\, dx}{\sqrt{1-x^2}}$
   (A) $x\ln|1-x^2|+C$ (B) $2\sqrt{1-x^2}+C$ (C) $\frac{-1}{2}\ln|1-x^2|+C$ (D) $-\sqrt{1-x^2}+C$
   (E) None of these

3. Evaluate $\frac{3x+1}{x^2+x}\, dx$
   (A) $6\ln|x+5|\ln|x+1|+C$ (B) $3\ln(x^2)+\ln|x|+C$ (C) $3\ln|x^2+x|+C$ (D) $\ln|x|-\ln|x+1|+C$ (E) $\ln|x+2|\ln|x+1|+C$

4. Evaluate $\int_1^3 \sqrt{x}\ln x\, dx$ (Give your answer correct to 2 decimal places.)
   (A) 1.94 (B) 1.50 (C) -0.21 (D) 1.01 (E) 1.27

5. Find the area of the region bounded by the graph of $y=\sin 2x$, the x-axis, and the lines $x=0$ and $x=\frac{\pi}{2}$.
   (A) 2 (B) 1 (C) 0 (D) $\frac{1}{2}$ (E) $\frac{3}{4}$

6. Find the area of the region bounded by the curves $x^2+4y=0$ and $x^2-4y-8=0$.
   (A) $\frac{2}{3}$ (B) $\frac{16}{3}$ (C) 6 (D) $\frac{4}{3}$ (E) $\frac{10}{3}$

7. Calculate the volume generated by revolving the region bounded by $y=\sqrt{x}$, the x-axis and $x=4$ about the y-axis. (Express your answer as a definite integral.)
   (A) $\pi\int_0^4 x\, dx$ (B) $\pi\int_0^4 \sqrt{x}\, dx$ (C) $2\pi\int_0^4 (4-x)\sqrt{x}\, dx$ (D) $2\pi\int_0^4 x^2\, dx$ (E) $2\pi\int_0^4 x^{3/2}\, dx$

8. Calculate the volume generated by revolving the area bounded by $y=\sqrt{x}$, the y-axis and $y=2$ about the x-axis. (Express your answer as a definite integral.)
   (A) $\pi\int_0^4 (2-\sqrt{x})^2\, dx$ (B) $\pi\int_0^4 (4-x)\, dx$ (C) $2\pi\int_0^4 (2-\sqrt{x})\, dx$ (D) $2\pi\int_0^4 (4\sqrt{x}-x)\, dx$
   (E) $\pi\int_0^4 x\, dx$

9. Calculate the centroid of a quarter circle of radius r.
   (A) $\bar{x}=\frac{r}{3\pi}, \bar{y}=\frac{r}{3\pi}$ (B) $\bar{x}=\frac{4r}{3}, \bar{y}=\frac{4r}{3}$ (C) $\bar{x}=\frac{4r}{\pi}, \bar{y}=0$ (D) $\bar{x}=\frac{4r}{\pi}, \bar{y}=\frac{4r}{\pi}$ (E) $\bar{x}=\frac{4r}{3\pi}, \bar{y}=\frac{4r}{3\pi}$

10. Calculate $\bar{x}$ (the x-coordinate of the centroid) of the region given in problem 7. Note the area of the region is $\frac{16}{3}$ square units.
    (A) $\bar{x}=2$ (B) $\bar{x}=\frac{3}{2}$ (C) $\bar{x}=\frac{9}{5}$ (D) $\bar{x}=\frac{12}{5}$ (E) $\bar{x}=\frac{1}{5}$

11. Find the work done in pumping the water out of the top of a cylindrical tank 5 ft in radius and 10 ft high, if the tank is initially half full of water, which weighs 62.4 lb/ft^3.
(A) $93,750\pi$ ft-lb (B) $58,550\pi$ ft-lb (C) $7,800\pi$ ft-lb (D) $15,600\pi$ ft-lb (E) None of these

12. A spring of natural length 12 ft requires a force of 6 lb to stretch it by 2 ft. Find the work done in stretching it by 6 ft.
(A) 54 ft-lb (B) 108 ft-lb (C) 6 ft-lb (D) 36 ft-lb (E) 24 ft-lb

13. A vertical rectangular floodgate on a dam is 5 ft long and 4 ft deep. Find the force on the floodgate if its upper edge is 3 ft below the water surface. (The weight density of water is 62.4 lb/ft^3.) Give your answer correct to the nearest integer.
(A) 7644 lb (B) 3900 lb (C) 1248 lb (D) 6240 lb (E) 2100 lb

14. A horizontal tank with vertical circular ends is filled with oil. If the radius of each end is 2 m, find the force on one end of the tank. (Assume w is the weight of the oil.) Express your answer as a definite integral. (Hint: Assume that the origin is at the center of one of the circular ends.)
(A) $2w \int_0^2 y\sqrt{4-y^2}\, dy$ (B) $w \int_{-2}^2 \sqrt{4-y^2}\, dy$ (C) $2w \int_{-2}^2 (2-y)\sqrt{4-y^2}\, dy$ (D) $2w \int_{-2}^2 (2-y)\, dy$ (E) None of these

15. Evaluate the improper integral $\int_4^\infty \frac{2}{x^2-1}\, dx$
(A) It is divergent (B) $\ln 4 - \ln 5$ (C) $\ln 5 - \ln 4$ (D) $\ln 3 - \ln 5$ (E) $\ln 5 - \ln 3$

16. $\sum_{n=1}^{\infty} \frac{3^{n-1}}{5^n} =$
(A) It is divergent (B) $\frac{5}{7}$ (C) $\frac{14}{29}$ (D) $\frac{5}{6}$ (E) $\frac{1}{2}$

17. Find the first three non-zero terms of the Maclaurin series of $f(x) = \sqrt{1+3x}$.
(A) $f(x) = 1 + \frac{3}{2}x - \frac{9}{4}x^2$ (B) $f(x) = 1 + \frac{1}{2}\sqrt{1+3x} - \frac{1}{8}(1+3x)$ (C) $f(x) = 1 + \frac{1}{2}x - \frac{1}{8}x^2$ (D) $f(x) = 1 + \frac{3}{2}\sqrt{1+3x} - \frac{9}{8}(1+3x)$ (E) $f(x) = 1 + \frac{3}{2}x - \frac{9}{8}x^2$

18. Using the Maclaurin series $\ln(1+x) = x - \frac{1}{2}x^2 + \frac{1}{3}x^3 - \frac{1}{4}x^4 + \frac{1}{5}x^5 - \cdots$, find the minimum number of terms required to calculate $\ln(1.3)$ so that the error is ≤ 0.001.
(A) 2 (B) 3 (C) 4 (D) 5 (E) 6

19. Find the first three non-zero terms in the Taylor series for $f(x) = \sin 2x$ in powers of $(x - \frac{\pi}{8})$.
(A) $f(x) = \sqrt{2}\left[\frac{1}{2} + (x-\frac{\pi}{8}) - (x-\frac{\pi}{8})^2\right]$ (B) $f(x) = 2(x-\frac{\pi}{8}) - \frac{3}{2}(x-\frac{\pi}{8})^2 + \frac{4}{15}(x-\frac{\pi}{8})^5$ (C) $f(x) = (x-\frac{\pi}{8}) - \frac{1}{3!}(x-\frac{\pi}{8})^3 + \frac{1}{5!}(x-\frac{\pi}{8})^5$ (D) $f(x) = \sqrt{2}\left[\frac{1}{2} + \frac{1}{2}(x-\frac{\pi}{8}) - \frac{1}{4}(x-\frac{\pi}{8})^2\right]$
(E) None of these.

20. Approximate $\int_0^{0.3} \cos\sqrt{x}\, dx$ using three terms of the appropriate Maclaurin series. Give your answer correct to 4 decimal places.
(A) 0.8538 (B) 0.2779 (C) 0.9553 (D) 0.2955 (E) 0.1863

21. If f is a periodic function of period 2π and

$$f(t) = \begin{cases} 0 & -\pi \le x < 0 \\ 1 & 0 \le x \le \frac{\pi}{2} \\ 0 & \frac{\pi}{2} < x \le \pi \end{cases}$$

calculate the first three non-zero terms of the Fourier series for $f(x)$. (A) $\frac{\pi}{4}+\cos x+\sin x$ (B) $\frac{1}{4}+\frac{1}{\pi}\cos x-\frac{1}{\pi}\sin x$ (C) $\frac{1}{4}-\frac{\sqrt{2}}{\pi}\cos x+\frac{1}{\pi}\cos 2x$ (D) $\frac{1}{4}+\frac{1}{\pi}\cos x+\frac{1}{\pi}\sin x$ (E) None of these.

22. Find the particular solution of the differential equation $y' + \dfrac{1}{x}y = x^2$ where $y = 2$ when $x = 1$.
(A) $y = \dfrac{x^4}{4} + \dfrac{7}{4}$ (B) $y = \dfrac{x^3}{3} + \dfrac{5}{3x}$ (C) $y = \dfrac{x^3}{4} + \dfrac{7}{4x}$ (D) $y = \dfrac{x^3}{4} + \dfrac{7}{4}$ (E) None of these.

23. Find the general solution of the differential equation $y^2\,dx + (x+1)^2\,dy = 0$
(A) $\frac{1}{3}(x+1)^3+\frac{1}{3}y^3 = C$ (B) $\frac{1}{x+1}+\frac{1}{y} = C$ (C) $\ln|x+1|+\ln|y| = C$ (D) $2(x+1)+2y = C$ (E) $x+\frac{1}{y} = C$

24. Find the equation of the curve for which the slope at any point (x, y) is $x+y$ and which passes through the point $(0, 1)$.
(A) $y = 2e^{-x} - x - 1$ (B) $y = \frac{1}{2}e^x + \frac{1}{2}x^2$ (C) $y = -x + 1$ (D) $y = 2e^x - x - 1$ (E) $y = e^x + 1$

25. Find the particular solution of the differential equation $y'' + y' - 6y = 0$ where $y' = 0$ and $y = -1$ when $x = 0$.
(A) $y = -\frac{1}{5}(2e^{-3x} + 3e^{2x})$ (B) $y = -\frac{1}{5}(2e^{3x} + 3e^{-2x})$ (C) $y = -\frac{1}{2}(e^{-3x} + e^{2x})$ (D) $y = -\frac{1}{2}(e^{3x} + e^{-2x})$ (E) None of these.

26. Find the general solution of the differential equation $y'' - y' + y = 0$.
(A) $y = c_1e^{(1+\sqrt{3})x/2} + c_2e^{(1-\sqrt{3})x/2}$ (B) $y = e^x[c_1\sin(\sqrt{3}x/2) + c_2\cos(\sqrt{3}x/2)]$ (C) $y = e^x[c_1\sin(\sqrt{3}x) + c_2\cos(\sqrt{3}x)]$ (D) $y = e^{x/2}[c_1\sin(\sqrt{3}x/2) + c_2\cos(\sqrt{3}x/2)]$ (E) None of these.

27. Find the general solution of the differential equation $y'' + 8y' + 16y = 0$.
(A) $y = c_1e^{-4x} + c_2xe^{-4x}$ (B) $y = c_1e^{4x} + c_2xe^{4x}$ (C) $y = c_1e^{-4x} + c_2e^{-4x}$ (D) $y = c_1\sin 4x + c_2\cos 4x$ (E) $y = c_1e^{-4x} + c_2e^{-4x}$

28. An object moves with simple harmonic motion according to the equation $\dfrac{d^2x}{dt^2} + 64x = 0$. Find the displacement x as a function of t if $x = 4$ and $\dfrac{dx}{dt} = 3$ when $t = 0$.
(A) $x = 4\sin 8t + \frac{3}{8}\cos 8t$ (B) $x = 3\sin 8t + 4\cos 8t$ (C) $x = \frac{3}{64}\sin 64t + 4\cos 64t$ (D) $x = \frac{3}{8}\sin 8t + 4\cos 8t$ (E) $x = 8\sin 8t + 4\cos 8t$

29. Find the Laplace transform of $2e^{-3t}\sin 4t$.

(A) $\dfrac{2}{(s-3)^2+16}$ (B) $\dfrac{8}{(s+3)^2+16}$ (C) $\dfrac{8}{(s-3)^2+16}$ (D) $\dfrac{8}{(s+3)(s^2+16)}$ (E) $\dfrac{2}{(s+3)^2+16}$

30. Find the inverse Laplace transform of $\dfrac{2s}{s^2+3s-4}$.

(A) $\frac{1}{10}(4e^{4t}-e^t)$ (B) $\frac{2}{5}(4e^{-4t}+e^t)$ (C) $\frac{1}{10}(4e^{4t}+e^t)$ (D) $\frac{2}{5}(4e^{4t}+e^{-t})$ (E) None of these.

31. Find the Laplace transform of the expression $y''-3y'+2y$, where $y(0)=-1$ and $y'(0)=2$. In the following choices the Laplace transform of $y(x)$ is denoted by $Y(s)$.
(A) $(s^2-3s+2)Y(s)$ (B) $s^2Y(s)+s-2$ (C) $(s^2-3s+2)Y(s)+s-1$ (D) $(s^2-3s+2)Y(s)+s+1$ (E) $(s^2-3s+2)Y(s)+s-5$

32. Find the Laplace transform of the solution of the differential equation $y'+2y=e^{-2t}$, $y(0)=2$.
(A) $\dfrac{1}{(s+2)^2}$ (B) $2+\dfrac{1}{s+2}$ (C) $\dfrac{2}{s+2}+\dfrac{1}{(s+2)^2}$ (D) $\dfrac{2}{s-2}+\dfrac{1}{(s-2)^2}$ (E) $\dfrac{1}{(s-2)^2}$

33. Use Laplace transforms to solve the differential equation $y''+9y=3t$, $y(0)=1$, $y'(0)=-1$.
(A) $y=\frac{1}{3}t-\frac{4}{9}\sin 3t+\cos 3t$ (B) $y=\frac{1}{9}t-\frac{10}{27}\sin 3t+\cos 3t$ (C) $y=4\cos 3t-\frac{1}{3}\sin 3t$ (D) $y=\cos 3t-\frac{1}{3}\sin 3t$ (E) None of these.

34. Use Laplace transforms to solve the differential equation $y''-2y'+y=e^t$, $y(0)=0$, $y'(0)=0$.
(A) $y=2t^2e^t$ (B) $y=\frac{1}{2}t^2e^{-t}$ (C) $y=\frac{1}{2}t^2e^t$ (D) $y=t^2e^{-t}$ (E) $y=2te^{-t}$

35. If $f(s)=\dfrac{s}{(s-1)^2(s+2)}$, which of the following is the partial fraction expansion of $f(s)$? In the following choices K_1, K_2, and K_3 are constants.
(A) $\dfrac{K_1}{s-1}+\dfrac{K_2}{s-1}+\dfrac{K_3}{s+2}$ (B) $\dfrac{K_1}{(s-1)^2}+\dfrac{K_2}{s+2}$ (C) $\dfrac{K_1}{s-1}+\dfrac{K_2}{(s-1)^2}+\dfrac{K_3s}{s+2}$ (D) $\dfrac{K_1}{s-1}+\dfrac{K_2}{(s-1)^2}+\dfrac{K_3}{s+2}$ (E) $\dfrac{K_1}{s-1}+\dfrac{K_2}{s+2}$

Answers
1. B 2. D 3. E 4. A 5. B 6. B 7. E 8. B 9. E 10. D 11. B 12. A 13. D 14. C 15. E 16. D 17. E 18. C 19. A 20. B 21. D 22. C 23. B 24. D 25. A 26. D 27. A 28. D 29. B 30. B 31. E 32. C 33. A 34. C 35. D