Mathematics of topological quantum computing

Topological quantum computing (TQC) is among the best approaches to building a large-scale fault-tolerant quantum computer. The quantum media for TQC are topological phases of matter that harbor non-Abelian anyons and quantum gates are implemented by braiding of anyons. The mathematics of topological phases of matter is described by modular tensor categories or equivalently by topological quantum field theories. We give a review on the rich interactions between TQC and the subjects mentioned above. We illustrate the concept of TQC with an important class of anyons, namely, metaplectic anyons, and show that braidings of anyons assisted by certain topologically protected measurements is universal for quantum computing. The interest in metaplectic anyons arises from the potential physical realization in fractional quantum Hall systems. Time permitting, we also talk about the application of topological quantum field theories in topology and give a new invariant of smooth 4-manifolds of state-sum type.