

DEPARTMENT OF MATHEMATICS
PURDUE UNIVERSITY
GRADUATE STUDENT HANDBOOK

Revised Fall 2021

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NOTE: Trailing double zeroes have been dropped from course numbers in this document, e.g., MA 54400 is abbreviated MA 544.

KEEP FOR FUTURE REFERENCE

<http://www.math.purdue.edu/academic/grad/handbook.pdf>

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THE GRADUATE PROGRAM

I. Introduction

This handbook contains regulations, requirements, and general information for various graduate programs in the Department of Mathematics. This supplements material in the Graduate Bulletin and in the Graduate School publication "Policies and Procedures Manual for Administering Graduate Student Programs". Any questions should be directed to the Graduate Office.

II. Advisors, Mentors, and Plan of Study

Each entering graduate student will be assigned an advisor to assist in the selection of a graduate program and registration for courses. The student will be expected in subsequent semesters to have this advisor give guidance in the program, unless, by mutual consent, another advisor is suggested to the Graduate Office. The list of course offerings should be consulted in planning a program since some courses are offered only once a year or less. Each degree program requires that a plan of study be submitted and approved by an advisory committee in the Department of Mathematics and by the Graduate School. A student in a Ph.D. program who has passed the Advanced Topics Examination (see section IV.(C)) will arrange for a new advisory committee of three or four faculty members, representative of the student's major area of interest. The chair usually becomes the student's thesis advisor. The expectation is that the students and their thesis advisors meet at least once a week during a fall/spring semester. A student must be registered as a candidate in the semester in which the degree is expected.

Each entering graduate student will be also assigned one additional faculty member as an informal mentor. The expectation is that the mentor guide the student until the student passes the Advanced Topic Exam, but many students continue to consult their mentors even later. The advisor helps and advises the student in choosing appropriate courses, planning for qualifying exams, and choosing a trajectory through the program.

The advisor and the student meet at least once a semester to plan for the next semester, and another time to assess the academic progress. The mentor program is meant to broaden exposure to the faculty, to provide support and advice on a variety of professional topics, to facilitate the transition into graduate school, and to improve the climate of the department. The mentor and the student are expected to meet several times during the semester to check on classes, progress towards the degree, professional aspirations, and life. Depending on student's needs and interests, the student may at any time switch advisor or mentor to another willing professor in the department, and any such change needs to be recorded in the Graduate Office.

At the end of every semester, professors file with the Graduate Office reports on their advisees, mentees, and the graduate students in their classes. In this way we can be more aware of our students' successes as well as of any issues an individual student may have and that can be served better with prompt addressing.

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III. Master of Science Programs

A plan of study must be submitted and approved by the department and the Graduate School before the semester in which the student expects to receive the degree. It is desirable that the plan of study be submitted electronically by the end of the second semester of graduate study. A plan of study will not be accepted if it contains courses with a large overlap in content. Subsequent requests for changes in the plan of study must be submitted electronically and approved by the advisor, Graduate Office, and the Graduate School. The Plan of Study Generator (POSG) may be accessed via the MyPurdue portal at <http://mypurdue.purdue.edu> (see section X). The master's advisory committee consists of faculty with whom a student has had classes or faculty approved by the Graduate Office.

The completion of an approved plan of study, with all grades of A or B with the possible exception of at most two grades of C, and a grade point average of at least 3.00 is required for the M.S. degree.

A minimum of 30 hours of course credits is required for the M.S. degree. The average time for completion of this degree is two years for a student holding a half-time assistantship and not transferring credits. For a half-time assistant whose performance is satisfactory, a time limit of four semesters and the intervening summer session is usually imposed for completion of the M.S. degree with continuation of departmental support.

The Computational Finance option requires 34 hours of course credits. See III. (D).

Examples of Plans of Study

- (a) For a student in pure mathematics, the plan of study should include complex analysis MA 530 (or 525), real analysis MA 504-544 (or 544-545), algebra MA 553-554 (or 557-558), topology MA 571 (or 572), and either logic MA 585 or one course for which some of the courses above are prerequisites.
- (b) For a student in applied mathematics who wishes to continue toward a Ph.D. degree after the M.S. degree, the plan of study should include complex analysis MA 530, real analysis MA 544, algebra MA 553-554, partial differential equations MA 523 (or MA 642 or 643), methods of applied mathematics MA 611 and numerical analysis (one of CS 514, 515, 614, 615).
- (c) For a student in applied mathematics not planning to continue on to the Ph.D. degree, the plan of study should include complex analysis MA 530 (or 525), real analysis MA 544 (or 504), linear algebra MA 554 (or 511), probability theory MA 519, partial differential equations MA 523 and numerical analysis CS 514 (or CS 515, or CS 614 or CS 615). In addition, at least two of the remaining four courses required for the degree should provide depth in a specific area. Possible options for these four courses are as follows:

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- (1) Mathematical Methods and Applications. Courses selected from mathematics (e.g. 533, 542, 611, 620, 626).
- (2) Numerical Analysis. MA 611 and courses selected from MA 524, CS 515, 614, 615, A & AE 516.
- (3) Statistics. Courses selected from STAT 528, 529, 532, 538, 553, 554, or 576 (statistical theory). Courses selected from STAT 512, 514, 520, 522, or 524 (applied statistics).
- (4) Discrete Mathematics. Courses selected from MA 518, 553, 575, 585, CS 580.
- (5) Operations Research. MA 521 and courses selected from MA 620, IE 535, 537, 538, MA/STAT 532.

Other combinations of courses are possible, and substitutions in course requirements and the transfer of credits from other universities may be permitted with the permission of the Graduate Committee.

- (d) Mathematics Degree with Specialization in Computational Finance (CF), 34 credit hours. Two-year intensive program.

Group I: Required Math Courses (excluding Math Finance) (15 credit hrs)

- Probability and Analysis: MA/STAT 519, and MA 544 or [MA 504 and MA 538]
- Linear Algebra: MA 554 or MA 511
- Partial Differential Equations: MA 523
- Methods of Applied Mathematics: 3 or more credits from the following list:
- MA 611, CS 514, CS 515, CS 614, CS 615, STAT 528 or STAT 525

Group II: Required Core CF courses (16 credit hours)

- Mathematics of Finance: MA 515
- Advanced Probability, Options, and Numerical Methods: MA 516
- Simulation Design and Analysis: IE 581 or Introduction to Computational Statistics STAT 598 G

and 7 or more credit hours approved by the CF committee from the following list:

- Financial Management I: MGMT 610 (3 cr.)
- Options and Futures: MGMT 641 (2 cr.)
- Security Analysis: MGMT 642 (2 cr.)
- Financial Risk Management: MGMT 643 (2 cr.)
- Portfolio Management: MGMT 614 (2 cr.)
- Spreadsheet Modeling and Simulation: MGMT 690S/570 (2 cr.)
- Seminar in Financial Markets: MGMT 616 (3 cr.)

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- Seminar in Financial Markets: MGMT 617 (3 cr.)
- Design and Analysis of Financial Algorithms: STAT 598 W (3 cr.)
- Venture Capital and Investment Banking: MGMT 644 (2 cr.)

Group III: Elective courses (3 credit hours)

3 or more credit hours of courses related to CF and approved by the CF committee. These can be courses from any department or school. The CF advisor will help students make a selection.

Possible departments to choose from include: Management, Economics, Statistics, Computer Science, Agricultural Economics, and Industrial Engineering. Possible topics include: Advanced finance seminar, Portfolio management, Security analysis, Macroeconomics, International monetary problems, Financial time series, Bayesian statistics in finance, and Monte-Carlo methods.

A typical degree plan for the MS degree in Mathematics with CF specialization will look like this:

MA 511, MA 519, MA 523, MA 544, STAT 525, MA 515, MA 516, IE 581, MGMT 610, MGMT 614, MGMT 643, ECON 608.

- (e) The following courses are NOT electives for the above programs: MA 510, 520, 527, 528, 560.

IV. Doctor of Philosophy Program

Besides satisfying the general regulations of the Graduate School for the degree of Doctor of Philosophy, the student must comply with the following requirements.

- (A) **Qualifying Examinations.** The student must pass four written examinations chosen as described below. The exams are based on material that is covered in the courses listed and on material from undergraduate prerequisites. Credit for passing a similar examination at another university cannot be transferred.

The Qualifying Examinations are written examinations offered twice a year during weeklong *Qualifier Exam Sessions* the week before classes start in August and January. Each examination is written and graded by a faculty member or a committee of faculty members chosen by the Graduate Committee.

The following four subject areas are called the **Core 4 Areas**.

Complex Analysis (MA 530)

Real Analysis (MA 544)

Abstract Algebra (MA 553)

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Linear Algebra (MA 554)

The qualifier exam subject areas are the Core 4 Areas plus the following **Area Exams**.

Numerical Analysis (MA 514)

Probability (MA 519)

Partial Differential Equations (MA 523)

Differential Geometry (MA 562)

Topology (MA 571)

The student must pass at least two exams from the Core 4 Areas, including at least one of 544 or 553. They must also pass two more exams from the Area Exams and the unused two exams from the Core 4.

The **Qualifier Deadline** for students who enter the program with a master's degree is the January Qualifier Exam Session of their second year. The Qualifier Deadline for students without a master's degree is the January Qualifier Exam Session of their third year. If a student reaches the deadline to complete their qualification examinations, and still needs to pass one exam, the Graduate Committee may accept a grade of B+ or better in a qualification exam course in place of the student passing a final qualification exam. Students who have not passed the four exams on or before the session of their Qualifier Deadline will have their privileges to continue in the Mathematics PhD program terminated.

Each Qualifier Exam can be attempted a maximum of three times and students may attempt as many Qualifier Exams as they wish at any Qualifier Session on or before their Qualifier Deadline. Qualifying exam grades will not be part of your transcript.

A syllabus for each area is given in section VI.

Previous exams are available at <http://www.math.purdue.edu/academic/grad/qualexams/>

(C) **Advanced Topics Examinations.** A student becomes eligible to take the Advanced Topics Examination after passing the Qualifying Examinations. The Advanced Topics Examination serves as the Preliminary Examination in the Department of Mathematics Ph.D. program.

Before taking the Advanced Topics Exam the student must have already passed all of their Core courses with a B- or better (or have passed the corresponding qualifying exam in lieu of taking the course). The Core 4 courses are: MA 530, 544, 553, 554.

After passing the Qualifying Examinations, a student must find a faculty member willing to serve as the Advanced Topics Examination Coordinator. This Coordinator, once identified, begins to serve as the student's academic advisor, counseling the student and signing course registration forms. Usually the Coordinator becomes the student's thesis advisor, provided the student passes the Advanced Topics Examination. The student must meet with the Coordinator

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to prepare an Advanced Topics Examination Proposal Form, which is to be filed in the Graduate Office at least one month prior to the Advanced Topics with the Graduate School.

The Advanced Topics Proposal form lists the Coordinator, two courses beyond the qualifying level on which the student is to be examined (or a body of mathematics roughly equivalent to this), and two other faculty members who, with the Coordinator, administers the Advanced Topics Examination. At the discretion of the Coordinator, the examination may also cover a third subject, possibly with a third examiner. The examinations may be oral or written, and may be given separately or together. The conditions of the examination are specified on the Advanced Topics Examination Proposal Form, which must be approved and signed by the Coordinator, the student, and the Graduate Committee Chair at least one month prior to the exam date. To pass the examination, requires agreement of all members of the committee and the consent of one to serve as the student's thesis advisor. While this is usually the Coordinator it need not be.

A student may take the Advanced Topics Examination at most twice; however, the examination should be passed within one and one half years of passing the Qualifying Examinations. In special cases the Graduate Committee may grant an extension of this time limit. Each time the examination is taken, a new Advanced Topics Examination Proposal Form must be filed in the Graduate Office.

Below is a list of some possible combinations of courses for the Advanced Topics Examination.

Algebra	(MA557, 558) (MA558, 650) (MA558, 664)
Analysis	(MA531, 631) (MA538, 545) (MA546, 646) (MA642, 643) (MA647, 648)
Applied Math	(MA611, 642) (MA642, 643)
Geometry	(MA562, 661)
Logic	(MA586, 587) (CS584, MA586)
Numerical Analysis	(CS614, 615)
Probability	(MA538, 539)
Topology	(MA572, 672)

- (D) **Plan of Study.** The plan of study should be submitted electronically to the Graduate School through MyPurdue by each student at least one month before passing the Advanced Topics Examinations. The student must arrange for an advisory committee to approve the plan of study, with the chair being the supervisor of thesis research. If you select a chair from outside the Mathematics Department then you must select a Mathematics faculty member to serve as a co-chair on your committee. This advisory committee must have at least half of its members with a faculty appointment over 50% in the Department of Mathematics.

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Substitutions in course requirements and the transfer of master's degree credits from other universities may be permitted with the approval of the advisory committee and the Graduate Committee. The Plan of Study Generator (POSG) may be accessed via the MyPurdue portal at <http://mypurdue.purdue.edu> (see section X).

The plan of study must include:

- (1) At least a total of 42 hours of Purdue graduate course work with at least half them in courses offered or co-listed by the Mathematics Department. However, all applicable courses (only those graduate courses with a letter grade) should be listed on the plan of study with the exception of any courses used for a Purdue Master's degree. A completed plan of study must list courses with at most two grades of C and all other grades of A or B. A total of 90 credit hours are required for the Ph.D. degree, but this total includes thesis research MA 699, which is not included on the plan of study.)
 - (2) At least three courses (nine credit hours) at an advanced level in the field of specialty or closely related to it. Reading courses and seminars may be included.
 - (3) The courses MA 530, 544, 553, and 554 and two courses from among MA 514, 519, 523, 562, 571, 572, and 585. A student can avoid taking any of these courses by passing the qualifier exam in the subject.
 - (4) For students in applied mathematics, at least two courses selected from CS 514, 515, 614, 615, and one course that uses advanced mathematics, taken outside the mathematical sciences.
 - (5) For students in numerical analysis, at least two courses selected from MA 523, 543, 611, 642, 643.
- (E) **Preliminary Examination.** The Advanced Topics Exam serves as the Preliminary Examination in the Math Ph.D. program. Graduate School regulations require that at least two sessions (including summer sessions) must elapse between the preliminary examination and the thesis defense. A request form must be submitted to the Graduate Office at least one month prior to the examination date. The Graduate office will fill this form out for you after you have submitted your Advanced Topics Proposal Form to the Graduate Office and your Plan of Study is on file with the Graduate School.

If a student has an advisor who is not in the Department of Mathematics then an advisory committee must be approved by the Graduate Committee of the Department of Mathematics. (For example, this could be the case for a student in the CS&E program.) This advisory committee must have at least half of its members with a faculty appointment over 50% in the Department of Mathematics. In this case, a preliminary examination may be required. The purpose of this exam is to ensure that the proposed thesis problem(s) is chiefly mathematical in nature, and that a thesis on this topic is appropriate as a thesis in the Department of Mathematics. A report on the preliminary examination shall be made in writing to the Graduate Committee

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of Mathematics discussing the proposed project, with particular emphasis on the mathematical content. The Graduate Committee will then make the final decision whether the thesis topic is acceptable.

- (F) **Admission to Candidacy.** To be admitted to candidacy for the Ph.D. degree, the student must fulfill the preceding requirements.
- (G) **Dissertation.** A thesis must be submitted in final form presenting new results of sufficient importance to merit publication. The student must distribute copies of their thesis to their committee for review at least three weeks before their defense is scheduled, or at the discretion of the student's advisor, the thesis defense can be postponed to a later date.

The student must present the contents of the thesis before an examining committee consisting of a minimum of four faculty members in an open colloquium or seminar. At least half of the examining committee must have a faculty appointment over 50% in the Department of Mathematics. The thesis must be acceptable to this committee. A request form for the appointment of the final examining committee must be received by the Graduate School not later than one month before the examination. The thesis must meet departmental and University format requirements. The Graduate Office will provide the necessary information.

- (H) **Recommendation for the Ph.D. Degree.** If the above requirements are met within the time limits stated below, the candidate will be recommended to the faculty to receive the degree of Doctor of Philosophy.
- (I) **Time Limits for Completion of the Ph.D. Degree.** The time limit for completion of a Ph.D. degree is set forth in the College of Science Policy and Procedures Manual, Section XVIII and reproduced here: 7 years from entry into the graduate program (i.e., 14 semesters plus the intervening summers plus one additional summer to finish if necessary) is the maximum time allowed to complete the Ph.D. in Science. An additional year may be allowed if requested by the student's Thesis Committee and approved by the Department's Graduate Studies Committee. Any exceptions to this policy shall require approval by the Department Head.
- (J) **Departmental Support for Graduate Students.** The Mathematics Department policy is to provide 6 years (12 academic year semesters) of financial support beginning with entry into the graduate program, contingent on adequate progress towards the Ph.D. and satisfactory performance of duties required for support (teaching, research, or other duties as assigned). Financial support during the intervening summers, plus one additional summer if necessary, is contingent on progress towards degree and performance of duties as indicated above, plus the availability of suitable positions within the Mathematics Department. An additional year of support may be allowed if requested by the student's Thesis Committee and approved by the Department's Graduate Studies Committee. Any extension beyond this time or any other exception to this policy requires approval from the Department Head.

(K) **Research in Absentia.** Research in absentia is possible only for students who have fulfilled the requirements (A) through (E) in section IV and are well into a research program. A request for permission from the Mathematics Department to do research in absentia requires the approval of the major professor, the Graduate Committee Chair, and the Graduate School. This form must be filed one month prior to the start of the session in which research in absentia registration is requested.

V. Computational Science and Engineering Program

The Computational Science and Engineering Program (CS&E) provides students with the opportunity to study a specific science or engineering discipline along with computing in a multidisciplinary environment. Participating departments include Aeronautics & Astronautics Engineering, Agricultural Economics, Agronomy, Biology, Chemistry, Computer Science, Earth, Atmospheric, and Planetary Sciences, Electrical Engineering, Food Science, Industrial & Physical Pharmacy, Mathematics, Mechanical Engineering, Medicinal Chemistry & Pharmacology, Nuclear Engineering, Pharmacy Practice, Physics, Psychology and Statistics.

For up to date information about this program, please consult the CS&E website at:
<https://www.purdue.edu/gradschool/cigp/index.html>.

Continued financial support by the Mathematics Department will depend on satisfactory academic progress and satisfactory performance in teaching and/or research duties (see section VII.). The Graduate Committee urges students to complete the Ph.D. within the term stated in their offer letter since financial support will be terminated after that time.

VI. Syllabus for Qualifying Examinations

Examinations will be based on material in references listed below for each area. Some topics may not be covered in courses listed in the previous subsection, IV. (A), in which case the student should study such topics in the suggested references. A list of the principal topics in each area is presented as an overview, but not as a detailed outline of the reference material. Previous exams are available at <http://www.math.purdue.edu/academic/grad/qualexams/>

(A) Real Analysis (MA 544)

Topics:

- a. Topology of metric spaces (properties of open, closed, compact, and connected sets).
- b. Continuity, semi-continuity, sequences of continuous functions and types of convergence, equicontinuity and Ascoli-Arzelà Theorem, Stone-Weierstrass theorem.

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- c. Construction of Lebesgue measure and integral, abstract measure spaces and integral, properties of measurable functions, L^p - spaces, Fubini-Tonelli theorem, convolutions in \mathbb{R}^n .
- d. Differentiation: Bounded variation and Helly Selection Theorem, Vitali Covering Theorem, differentiation of monotone functions, absolute continuity, Lebesgue Differentiation Theorem.

Possible books:

For (a) and (b)

W. Rudin, Principles of Mathematical Analysis – Chapters 2, 3, 4, 7

For (c)

A. Torchinsky, Real Variables – Chapters 4, 5, 6, 7, 12, 13

H. L. Royden, Real Analysis – Chapters 3, 4, 6, 11, 12.4

R.L. Wheeden and A. Zygmund, Measure and Integral – Chapters, 3, 4, 5, 6, 8.1-8.4, 9.1-9.2, 10.1-10.3

For (d)

A. Torchinsky, Real Variables – Chapters 8.2, 10

H.L. Royden, Real Analysis, Chapters 5.1-5.4

R.L. Wheeden and A. Zygmund, Measure and Integral, Chapters 2.1, 7.1-7.5

(B) Abstract Algebra (MA 553)

Prerequisites:

Some undergraduate level linear algebra and group theory such as is found on pages 1-99 of M. Artin's Algebra and, in addition, D&F (see below), Chapters 0-3 (except §3.4).

Topics:

- (a) Group theory; Sylow theorems; Jordan-Hölder theorem; solvable groups. [D&F, Chapters 4-5, plus §3.4].
- (b) Ring theory; unique factorization in polynomial rings and principal ideal domains. [D&F, Chapters 7-9].
- (c) Field theory; ruler and compass constructions, roots of unity, finite fields, Galois theory; solvability of equations by radicals. [D&F, Chapters 13-14, up to and including 14.7].

Possible book:

[D&F]: D. Dummit and R. Foote, *Abstract Algebra*, 2nd. Edition.

(B) Complex Analysis (MA 530)

Topics:

- (a) Cauchy-Riemann equations; conformality and other properties of analytic functions; linear fractional transformations; special functions.

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- (b) Taylor and Laurent series; absolute and uniform convergence.
- (c) Cauchy's theorem, formula, residue theorem, inequality; Morera's theorem; classification of singularities; Liouville's theorem; fundamental theorem of algebra; Casorati-Weierstrass theorem; definite integrals; maximum modulus theorem; Schwarz's lemma; Rouché's theorem; Weierstrass' theorem.
- (d) Harmonic functions; Poisson formula, Schwarz's theorem; harmonic conjugates; reflection principle.

Possible book:

Ahlfors: *Complex Analysis*, 3rd Edition, pp. 1-48, 67-84, 101-186.
Stein and Shakarchi: *Complex Analysis*, pp. 1-60, 64-110, 205-230.

(C) Linear Algebra (MA 554)

Topics:

- (a) Vector spaces; linear maps; matrices; determinants; systems of linear equations.
- (b) Inner products; hermitian, unitary and normal operators.
- (c) Modules over a principal ideal domain; finitely generated abelian groups; Jordan and rational canonical forms for a linear transformation.

Possible books:

Hoffman and Kunze, *Linear Algebra*, Chapters 1-8 (omitting §§5.6, 5.7)
Jacobson, *Basic Algebra I*, Chapter 3 (omit §3.11)

(D) Differential Geometry (MA 562)

Prerequisites: Some undergraduate multivariate calculus and topology as found in Munkres, Chapters 1-4 (see below) including the topology of \mathbb{R}^n , the chain rule for mappings from \mathbb{R}^n into \mathbb{R}^m , the implicit and inverse function theorems, and Jacobians.

Topics:

- (a) Differentiable manifolds and submanifolds; differentiable mappings, rank of a mapping and immersions, submanifolds, tangent and cotangent bundles.
- (b) Vector fields, Lie groups, One parameter groups, Lie bracket, Frobenius' theorem.
- (c) Tensors and tensor fields on manifolds; exterior algebra, orientation, integration on manifolds, Stokes' Theorem on manifolds.

Possible books:

J. Munkres, *Analysis on Manifolds*, Chapters 1-4.
W. Boothby, *Differentiable Geometry and Riemannian Geometry*, Chapters 1-6.

(F) Probability (MA 519)

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Topics:

- (a) Probability spaces and axioms; countable additivity of probability laws.
- (b) Combinatorial analysis.
- (c) Discrete random variables.
- (d) Continuous random variables.
- (e) Jointly distributed random variables; distributions of functions of random variables.
- (f) Expectations, variance, moments.
- (g) Jointly normal random variables in detail.
- (h) Limit theorems (e.g., weak law of large numbers and especially the central limit theorem).

Possible books:

Hoel, Port, and Stone Introduction to Probability, Chapters 1-7, (omitting starred sections, except 5.4 and 6.7; Some of Chapter 8)

Supplements to:

Multidimensional Changes of Variables:

- (i) Stroock, A Concise Introduction to the Theory of Integration, Section IV.3
- (ii) Papoulis, Probability, Random Variables and Stochastic Processes, Sections 7.1 and 7.2

Jointly normal random variables: see Breiman, *Probability*: With a view towards applications

Central Limit Theorem: see, for example, Breiman, *Probability* Chapter 1 and section 8.6 (pp. 167-170)

(G) Introduction to Partial Differential Equations (MA 523)

Topics:

- (a) Integral curves and surfaces of vector fields; Quasi-linear and linear equations of first order.
- (b) Characteristics; classification; canonical forms.
- (c) Separation of variables; Sturm-Liouville problems; Fourier series and convergence theorems.
- (d) Equations of mathematical physics; Laplace equation; wave equation; heat equation.
- (e) Cauchy-Kowalewski theorem; Holmgren Uniqueness theorem.

Possible books:

Churchill and Brown, *Fourier Series and Boundary Value Problems*, 4th Edition, Chapters 2, 3, 4.

Zachmanoglou and Thoe, Introduction to Partial Differential Equations, Chapters 2, 3, 4, 5-10.

John, Partial Differential Equations.

Chapter 1: §§1-6

Chapter 2: §§1-4

Chapter 3: §§1-6

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Chapter 4: §§1-3

Chapter 5: §1

Chapter 7: §1

(H) Topology (MA 571)

Topics:

- (a) Topological spaces and continuous functions, basis for a topology, product topology (for finite and infinite products), subspace topology, topology induced by a metric.
- (b) Connectedness, path connectedness, local connectedness. Compactness and local compactness. The compact-open topology.
- (c) Homotopy of paths and the fundamental group. Covering spaces. Fundamental groups of some important spaces (circle, sphere, torus, projective space).
- (d) Free products of groups. Statement of the Seifert-van Kampen theorem (but not the proof). Use of Seifert-van Kampen to calculate the fundamental groups of various spaces. Classification of Surfaces.

Possible books:

Munkres, *Topology*, Second Edition. Chapter 2 (including the starred sections and the supplementary exercises), Chapter 3 (including the starred sections but not the supplementary exercises), Section 46, Chapter 9 (not including the starred sections), Chapter 11, and Chapter 12.

(I) Numerical Analysis (CS/MA 514)

Topics:

- (a) Machine Arithmetic, Error Propagation and the Conditioning of Problems: real numbers, machine numbers, rounding; machine arithmetic; propagation of rounding errors, cancellation errors; conditioning of problems, examples.
- (b) Approximation and Interpolation: least squares approximation and data fitting; orthogonal polynomials; polynomial interpolation, Lagrange's formula; interpolation error and convergence; interpolation at Chebyshev points, Chebyshev polynomials; Newton's form of the interpolation polynomial; Hermite interpolation; inverse interpolation; interpolation by means of spline functions, minimal properties of spline interpolant's.
- (c) Numerical Differentiation and Integration: finite difference approximation of derivatives; numerical integration by composite trapezoidal and Simpson rules; Newton-Cotes formulae; Gaussian quadrature formulae; approximation of linear functionals, methods of interpolation and undetermined coefficients; extrapolation methods, Romberg integration.
- (d) Nonlinear Equations: examples; iterative methods, order of convergence; bisection method; secant method and its convergence properties; Newton's method, local and global convergence; algebraic equations; systems of nonlinear equations (briefly).
- (e) Ordinary Differential Equations: one-step methods, local and global error; Runge-Kutta methods; stiff equations; multistep methods.

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Possible books:

G. Dahlquist & A. Björck, *Numerical Methods*

J. Stoer & R. Bulirsch, *Introduction to Numerical Analysis*

VII. Policy Statements on Teaching and Academic Progress

- (A) Decisions on yearly salaries of graduate teaching assistants are based on a system of steps determined by years of service and progress toward an advanced degree. Promotion to higher steps is contingent upon satisfactory performance in the following areas as determined by the Associate Head in consultation with the Graduate Teaching Program Director and the Associate Head of Graduate Studies:
- (1) Quality of teaching as determined from evaluations by the lecturer and students.
 - (2) Quality of paper grading as determined from the professor's evaluation.
 - (3) Progress toward more advanced teaching assignments.
 - (4) Course grades.
 - (5) Satisfactory academic progress (see (D) below).
- (B) First year students whose oral English is satisfactory are usually assigned to teach 2 recitation sections (4 contact hours per week) of a large lecture course and to grade papers in that course. Advanced students are expected to be able to eventually teach their own courses. Students whose oral English is unsatisfactory will be assigned as paper graders in their first year. However, in order for these students to eventually be able to assume teaching duties, they will also be assigned to a special course in English during their first semester. This assignment will be continued until their oral and written English is satisfactory. **If oral English is inadequate for a classroom assignment at the end of one year, the assistantship will be terminated.**
- (C) Master's students are generally not supported by the department.
- (D) **Satisfactory Academic Progress.** Students are expected to maintain a cumulative GPA of at least a 3.0 as master's candidates and at least a 3.25 as Ph.D. candidates. Students must show active progress towards a degree. Ph.D. students are expected to progress through the Qualifying Exams and Advanced Topics Exams in a timely manner (see sections IV(A) and IV(C)). Teaching assistantships and fellowships may be affected for those students with unsatisfactory academic progress. Students with continued unsatisfactory academic progress may have their financial support and also their privileges to continue in the mathematics Ph.D. program terminated.

VIII. Miscellaneous

- (A) PUID's and Computer accounts: A PUID may be obtained from Room 14 in the Hovde Hall of Administration. A picture ID is required and you must know your 10 digit PUID. All mathematics graduate students receive a departmental computer account. Additionally,

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students are assigned a career account by Purdue University. Students must use their career account to access myPurdue.purdue.edu for course registration, etc.

- (B) Computer Facilities: The Department maintains a network of Sun workstations and equipment for high quality graphics output. All graduate student offices contain workstations and there are workstations and printers in computer labs on each floor for graduate student use. The computer consultants maintain a Frequently Asked Questions (FAQ) webpage at: <http://www.math.purdue.edu/resources/computing/faq>
- (C) Supplies: Graduate students may obtain paper, pens, and pencils as needed. Paper clips and rubber bands are available from the Math Main Office, room 835, as are envelopes and letterhead for mathematics-related correspondence. Colored chalk, transparencies and pens for classroom use with overhead projectors are also available.
- (D) Research Awards: Each year, the Graduate Committee chooses the recipients of several research awards following the wishes of the donors and the Purdue rules.
- (E) Teaching awards: The department presents monetary awards each year to mathematics graduate students whose teaching has been exemplary. Nominations are solicited and recipients are chosen by a faculty committee.
- (F) Each year in early September the department will look at the Mathematics graduate students (and TAs from other departments teaching for us) who have taught some of the most difficult courses available for them in the Department and who have done so very successfully. Those whose records warrant it will thereafter have the title "Outstanding Graduate Instructor."
- (G) Purdue University encourages people who believe that they have experienced or witnessed sexual harassment to seek assistance within the University. The University offers both informal and formal procedures for dealing with complaints involving sexual harassment. Information is available in the Graduate Office.
- (H) Students with disabilities: If you are a person with a disability and may require a reasonable accommodation (e.g. modified or special equipment, adjustments to your teaching or class schedule, modified teaching materials, etc.) to enable you to perform your duties as a T.A., please contact Dr. Dominic Naughton (MATH 826, 494-1906).
- (I) The homepage address for the department is <http://www.math.purdue.edu/>. Much information about the department, seminars and related off-campus links may be found there.
- (J) The Graduate Office maintains a Frequently Asked Questions (FAQ) webpage addressing many questions and issues. Go to <http://www.math.purdue.edu/academic/grad/faq>

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IX. Graduate Math Courses

The department offers a wide range of graduate courses in a large variety of areas of mathematics. The following is a sample. The current list can be found here:

<https://www.math.purdue.edu/academic/courses/Course-Description.html>.

The graduate courses have numbers >500. In addition to the regular course offerings, numerous advanced topics and seminars are given each year.

General Courses

503-Abstract Algebra. *Prerequisite:* two upper-division mathematics courses, one on linear algebra and one on abstract algebra.

Group theory: definitions, examples, subgroups, quotient groups, homomorphisms, and isomorphism theorems. Ring theory: definitions, examples, homomorphisms, ideals, quotient rings, fraction fields, polynomial rings, Euclidean domains, and unique factorization domains. Field theory: algebraic field extensions, straightedge and compass constructions.

504-Real Analysis. *Prerequisite :* two upper-division mathematics or engineering courses.

Completeness of the real number system, basic topological properties, compactness, sequences and series, absolute convergence of series, rearrangement of series, properties of continuous functions, the Riemann- Stieltjes integral, sequences and series of functions, uniform convergence, the Stone-Weierstrass Theorem, equicontinuity, the Arzela-Ascoli Theorem.

510-Vector Calculus. *Prerequisite:* MA 262 or 272. Not open to students with credit in MA 362 or 410.

Calculus of functions of several variables and of vector fields in orthogonal coordinate systems; optimization problems; the implicit function theorem; Green's, Stokes', and the divergence theorems; applications to engineering and the physical sciences.

511-Linear Algebra with Applications. *Prerequisite:* MA 262

Real and complex vector spaces; linear transformations; Gram-Schmidt process and projections; unitary and orthogonal diagonalization; Jordan canonical form; quadratic forms.

514-Numerical Analysis. *Prerequisite:* Authorized equivalent courses or consent of instructor may be used in satisfying course pre- and co-requisites. (CS 514) Iterative methods for solving nonlinear; linear difference equations, applications to solution of polynomial equations; differentiation and integration formulas; numerical solution of ordinary differential equations; roundoff error bounds.

518-Advanced Discrete Mathematics. *Prerequisite:* MA 262 or equivalent or consent of instructor.

The course covers mathematics useful in analyzing computer algorithms. Topics include recurrence relations, evaluation of sums, integer functions, elementary number theory, binomial coefficients, generating functions, discrete probability, and asymptotic methods.

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519-Intro to Probability (STAT 519). *Prerequisite:* MA 510; or co-requisite: MA 341 or 440.

Algebra of sets, sample spaces, combinatorial problems, independence, random variables, distribution functions, moment generating functions, special continuous and discrete distributions, distribution of a function of a random variable, limit theorems.

520-Boundary Value Problems of Differential Equations. *Prerequisite:* MA 303 or 304, or equivalent.

Separation of variables; Fourier series; boundary value problems; Fourier transforms; Bessel functions; Legendre polynomials.

523-Intro to Partial Differential Equations. *Prerequisite :* MA 266 or 366, MA 440 and MA 362 or 410 or 510.

First order quasi-linear equations and their application to physical and social sciences; the Cauchy-Kovalevsky theorem; characteristics, classification, and canonical form of linear equations; equations of mathematical physics; study of the Laplace, wave and heat equations; methods of solution.

525-Intro to Complex Analysis. *Prerequisite:* MA 362, 410, or 510.

Complex numbers and complex-valued functions of one complex variable; differentiation and contour integration; Cauchy's theorem; Taylor and Laurent series; residues; conformal mapping; applications.

527-Advanced Mathematics for Engineers and Physicists I. *Prerequisite:* MA 262; MA 511 is recommended.

MA 527 and 528 constitute a two-semester sequence covering a broad range of subjects useful in early graduate engineering courses. Topics in MA 527 include linear algebra, systems of ordinary differential equations, Laplace transforms., Fourier series and transforms, and partial differential equations.

528-Advanced Mathematics for Engineers and Physicists II. *Prerequisite:* MA 262; MA 510 is recommended.

MA 527 and 528 constitute a two-semester sequence covering a broad range of subjects useful in early graduate engineering courses. Topics in MA 528 include divergence theorem, Stokes' theorem, complex 12 variables, contour integration, calculus of residues and applications, conformal mapping, and potential theory.

530-Functions of a Complex Variable I. *Prerequisite or co-requisite:* MA 544. (More mathematically rigorous than MA 525).

Complex numbers and complex-valued functions of one complex variable; differentiation and contour integration; Cauchy's theorem; Taylor and Laurent series; residues; conformal mapping; special topics.

531-Functions of a Complex Variables II. *Prerequisite:* MA 530.

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Advanced topics.

532-Elements of Stochastic Processes (STAT 532). *Prerequisite:* MA 519.

A basic course in stochastic models, including discrete and continuous time Markov chains and Brownian motion, as well as an introduction to topics such as Gaussian processes, queues, epidemic models, branching processes, renewal processes, replacement, and reliability problems.

538-Probability Theory I (STAT 538). *Prerequisite:* MA 544.

Mathematically rigorous, measure-theoretic introduction to probability spaces, random variables, independence, weak and strong laws of large numbers, conditional expectations and martingales.

539-Probability Theory II (STAT 539). *Prerequisite:* MA 530 and 538.

Convergence of probability laws, characteristic functions; convergence to the normal law; infinitely divisible and stable laws; Brownian motion and the invariance principle.

542-Theory of Distributions and Applications. *Prerequisite:* MA 510 and 525 or equivalent.

Definition and basic properties of distributions: convolution and Fourier transforms; applications to partial differential equations; Sobolev spaces.

543-Intro to the Theory of Ordinary Differential Equations. *Prerequisite:* MA 361.

Existence and uniqueness theorems for ordinary and functional differential equations; linear theory; self-adjoint problems; nonlinear and perturbation theory.

544-Real Analysis and Measure Theory. *Prerequisite:* MA 442 or 504.

Metric space topology; continuity, convergence; equicontinuity; compactness; bounded variation, Helly selection theorem; Riemann-Stieltjes integral; Lebesgue measure; abstract measure spaces; L_p -spaces; Holder and Minkowski inequalities; Riesz-Fischer theorem.

545-Functions of Several Variables and Related Topics. *Prerequisite:* MA 544.

Differentiation of functions; Besicovitch covering theorem; differentiation of one measure with respect to another; Hardy-Littlewood maximal function; functions of several variables; Sobolev spaces.

546-Intro to Functional Analysis. *Prerequisite:* MA 544.

Fundamentals of functional analysis; Banach spaces. Hahn-Banach theorem; principle of uniform boundedness; closed graph and open mapping theorem; applications; Hilbert spaces; orthonormal sets; spectral theorem for Hermitian operators and for compact operators.

553-Intro to Abstract Algebra. *Prerequisite:* MA 453.

Group theory: Sylow theorems, Jordan-Hölder theorem, solvable groups. Ring theory unique factorization in polynomial rings and principal ideal domains. Field theory: ruler and compass constructions, roots of unity, finite fields, Galois theory, solvability of equations by radicals.

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554-Linear Algebra. *Prerequisite:* MA 350 or equivalent.

Review of basics: vector spaces; dimension; linear maps; matrices; determinants; linear equations. Bilinear forms; inner product spaces; spectral theory; eigenvalues. Modules over a principal ideal domain; finitely generated abelian groups; Jordan and rational canonical forms for a linear transformation.

557-Abstract Algebra I. *Prerequisite:* MA 454.

Review of fundamental structures of algebra (groups, rings, fields, modules, algebras); Jordan-Hölder and Sylow theorems; Galois theory; bilinear forms; modules over principal ideal domains; Artinian rings and semisimple modules; Polynomial and power series rings; Noetherian rings and modules; localization; integral dependence; rudiments of algebraic geometry and algebraic number theory; ramification theory.

558-Abstract Algebra II. *Prerequisite:* MA 557.

A continuation of MA 557. This course is usually an introduction to commutative algebra. Noetherian rings and modules, localization, integral dependence, Going Up and Going Down Theorems, Hilbert's Basis Theorem and Nullstellensatz, Noether Normalization Theorem, and Primary Decomposition.

560-Fundamental Concepts of Geometry. *Prerequisite:* MA 261.

Foundations of Euclidean geometry, including a critique of Euclid's "Elements" and a detailed study of an axiom system such as that of Hilbert. Independence of the parallel axiom and introduction to non-Euclidean geometry.

562-Intro to Differential Geometry and Topology. *Prerequisite:* MA 351 and 442.

Smooth manifolds; tangent vectors; inverse and implicit function theorems; submanifolds; vector fields; integral curves; differential forms; the exterior derivative; DeRham cohomology groups; surfaces in E^3 ; Gaussian curvature; two-dimensional Riemannian geometry; Gauss-Bonnet and Poincaré theorems on vector fields.

571-Elementary Topology. *Prerequisite:* MA 440 or 504.

Fundamentals of point set topology with an introduction to the fundamental group and related topics: topological spaces, product topology, quotient topology, compactness and connectedness, function spaces, homotopy of paths and the fundamental group, covering spaces, Seifert-van Kampen theorem.

572-Intro to Algebraic Topology. *Prerequisite:* MA 571.

Singular homology theory; Eilenberg-Steenrod axioms; simplicial and cell complexes; elementary homotopy theory; Lefschetz fixed point theorem.

575-Linear Graph Theory. *Prerequisite:* MA 351 or equivalent.

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Introduction to graph theory with applications.

584-Algebraic Number Theory. *Prerequisite:* MA 553, 554. Authorized equivalent courses or consent of instructor may be used in satisfying course pre- and co-requisites.

Dedekind domains, norm, discriminant, different, finiteness of class number, Dirichlet unit theorem, quadratic and cyclotomic extensions, quadratic reciprocity, decomposition and inertia groups, completions and local fields.

585-Mathematical Logic I. *Prerequisite:* MA 385 or 453.

Propositional and predicate calculus; the Gödel completeness and compactness theorems; primitive recursive and recursive functions; the Gödel incompleteness theorem; Tarski's theorem; Church's theorem; recursive undecidability, special topics such as nonstandard analysis.

598-Topics in Mathematics

Supervised reading courses as well as dual-level special topics courses are given under this number.

611-Methods of Applied Mathematics I. *Prerequisite:* MA 511 or equivalent and MA 544.

Banach and Hilbert spaces; linear operators; spectral theory of compact linear operators; applications to linear integral equations and to regular Sturm-Louville problems for ordinary differential equations.

615-Numerical Methods for Partial Differential Equations I (CS 615). *Prerequisite:* Prerequisite: MA 514, 523. Authorized equivalent courses or consent of instructor may be used in satisfying course pre- and co-requisites.

Finite element method for elliptic partial differential equations; weak formulation; infinite-dimensional approximations; error bounds; algorithmic issues; solving sparse linear systems; finite element method for parabolic partial differential equations; backward difference and Crank-Nicholson time-stepping; introduction to finite difference methods for elliptic, parabolic, and hyperbolic equations; stability, consistency, and convergence; discrete maximum principles.

620-Mathematical Theory of Optimal Control. *Prerequisite:* MA 544

Existence theorems; the maximum principle; relationship to the calculus of variations; linear systems with quadratic criteria; applications.

631-Several Complex Variables. *Prerequisite:* MA 530.

Power series, holomorphic functions, representation by integrals, extension of functions holomorphically to convex domains. Local theory of analytic sets (Weierstrass preparation theorem and consequences). Functions and sets in the projective space P^n (theorems of Weierstrass and Chow and their extensions).

638-Stochastic Processes I (STAT 638). *Prerequisite:* MA 539.

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Advanced topics in probability theory which may include stationary processes, independent increment processes, Gaussian processes; martingales, Markov processes, ergodic theory.

639-Stochastic Processes II (STAT 639)

Continuation of MA 638.

642-Methods of Linear and Nonlinear Partial Differential Equations I. *Prerequisite:* MA 523 and 611.

Second order elliptic equations including maximum principles, Harnack inequality, Schauder estimates, and Sobolev estimates. Applications of linear theory to nonlinear equations.

643-Methods of Linear and Nonlinear Partial Differential Equations II. *Prerequisite:* MA 642.

Continuation of MA 642. Topics to be covered are: L^p theory for solutions of elliptic equations including Moser's estimates, Aleksandrov maximum principle, Calderon-Zygmund theory. Introduction to evolution problems for parabolic and hyperbolic equations including Galerkin approximation and semigroup methods. Applications to nonlinear problems.

644-Calculus of Variations. *Prerequisite:* MA 544.

Direct methods; necessary and sufficient conditions for lower semicontinuity of multiple integrals; existence theorems and connections with optimal control theory.

650-Commutative Algebra. *Prerequisite:* MA 558.

The study of those rings of importance in algebraic and analytic geometry and algebraic number theory. Topics are at the discretion of the instructor. Possible topics include regular rings and Cohen-Macaulay rings, homological algebra, flatness, Gorenstein rings, projective dimension.

661-Modern Differential Geometry. *Prerequisite:* MA 544, 554.

Differential manifolds, tangent vectors, vector fields and differential forms, tensor fields, DeRham's theorems, imbedding theorems, Riemannian geometry, curvatures, harmonic integrals.

663-Algebraic Curves and Functions I. *Prerequisite:* MA 558.

Algebraic functions of one variable from the geometric, algebraic, or function-theoretic points of view. Riemann-Roch theorem, differentials.

664-Algebraic Curves and Functions II. *Prerequisite:* MA 663.

Continuation of MA 663. Topics chosen by the instructor.

665-Algebraic Geometry. *Prerequisite:* MA 650 or 663.

Topics of current interest will be chosen by the instructor.

684-Class Field Theory. *Prerequisite:* Prerequisite: MA 584. Authorized equivalent courses or consent of instructor may be used in satisfying course pre- and co-requisites.

Ideles, adeles, L-functions, Artin symbol, reciprocity, local and global class fields, Kronecker-Weber Theorem.

690-Topics in Algebra

691-Topics in Logic and Foundations

692-Topics in Applied Mathematics

693-Topics in Analysis

694-Topics in Differential Equations

696-Topics in Geometry

697-Topics in Topology

699-Research Ph.D. Thesis

X. How to file a Plan of Study

Each graduate student admitted to a degree program must file a Plan of Study (POS). A formal Plan of Study should be created as early as feasible in the student's career because it guides a student's academic degree progress. plan of study is an academic contract between a student, the faculty members of the advisory committee, and the Graduate School. All departmental and Graduate School policies related to the filing of a Plan of Study must be adhered to explicitly.

Students filing their plan of study should complete their plan electronically. Access to the electronic Plan of Study Generator (POSG) is via the myPurdue portal at <http://mypurdue.purdue.edu>. It is necessary to login with your Career Account User ID and password to proceed. Once you have logged in, information pertinent specifically to graduate students is located under the Academic tab, and in the Graduate Students box on the lower right. The Graduate School provides access to the POSG though the Graduate School Plan of Study link.

Once you have clicked on the Graduate School Plan of Study link within myPurdue, a new browser window will open with the POSG login screen. This browser window is now separate from the myPurdue window. It is necessary to login with your Career Account User ID and password to proceed. Once you have logged in, the POSG links are available to you.

To begin your plan of study, click on the Plan of Study Generator link, and then click on "Create new plan of study" link. Once in the POSG, refer to the Help buttons located on each page to assist you in using the electronic POSG. You do not need to complete the entire form in one sitting; you may save your plan of study and return to it later. You may not bookmark any pages within the Graduate School link. To return to the POSG you must login to myPurdue.

When you have completed your plan of study and feel it is ready for review of your advisory committee, submit your plan as a **Draft**. All plans of study must first be submitted as Draft before you can submit your plan as a Final. While your plan is in Draft status, review the information with your advisory committee and your departmental coordinator to ensure that it satisfies department and Graduate School policies. Use your draft as a basis to discuss your academic and research goals with your advisory committee members. Once your entire committee has verbally

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accepted your plan of study, return to the POSG and submit your plan as "Final." The plan of study form will be electronically routed, reviewed and, if approved, signed by the departmental coordinator, your advisory committee and the Graduate School. You may check the status of your plan at any time by returning to the POSG and clicking on the Display Submitted Plan of Study link.

Once the Graduate School has approved your plan of study, you should check it every semester to monitor your academic degree progress.

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