

Erratum for

EISENSTEIN SERIES AND AUTOMORPHIC L -FUNCTIONS

(Compiled by Abhishek Parab)

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This note is an addendum to the book “Eisenstein Series and Automorphic L -functions” by Prof. Freydoon Shahidi. Additionally we have compiled a list of misprints or typos. If you find errors or misprints you may email them to me at abhishekparab@gmail.com. In the following, “Line εx ” will denote line $\#x$ from the top or bottom depending on the sign ε counting equations as one line but excluding the header. Any errors in this erratum are entirely my fault!

1 An Important Comment on Section 8

The theory of γ -factors developed in the method as surveyed in Section 8 and in particular in Theorem 8.3.2, plays an important role in Arthur’s theory of “Local Intertwining Relations” (LIR) and thus his endoscopic transfer from classical groups to $GL(n)$, our reference [A9]. In a recent manuscript [AGIKMS24], the authors devote a section, Section 2.6, making a detailed discussion of the method and our references [Sh1, Sh3, Sh5, Sh6, Sh8, KeSh], in an effort to make them agreeable to theirs. While they make several observations of what changes must be made, they give no root causes for this discrepancy. Let us now explain what the root cause is and that our results are in complete agreement with theirs. In fact, if our map $H_T : T(k) \longrightarrow X_*(T)_k$ given in page 23 of the book is changed to its negative, which is very likely what their H_T is, the formula in line -6 of Theorem 8.3.2 will change to

$$(1) \quad C_\chi(s\tilde{\alpha}, \sigma, w_0) = \lambda_G(\psi_k, w_0)^{-1} \prod_{i=1}^m \gamma_i(is, \sigma, \psi_k, w_0)$$

which is exactly their equation (S1) in Page 48 of [AGIKMS24]. (See also the comment below about ψ_k .) Our choice of H_T goes back to Langlands in his Euler Products manuscript [La1] which we consistently used in our papers and contributions to the method, using \tilde{r}_i in our γ -factors as he did. As soon as our H_T is replaced by its negative:

$$(2) \quad q^{-\langle H_T(t), \chi \rangle} = |\chi(t)|_k,$$

\tilde{r}_i appearing in the γ -factors will change to r_i as in [AGIKMS24] and thus their formula (S1). (The normalization (2) agree's with Cartier's article in his Corvallis lectures.) We refer to our reference [Sh20], page 310, where the choice of \tilde{r} as opposed to r is justified as a remark.

The conjugation $\bar{\psi}_k$ of ψ_k in line -6 of Theorem 8.3.2 is a typo and should be ψ_k . In fact, what appears in Proposition 3.4 of [KeSh] together with the results in [Sh6], which are the main tools in obtaining the formula in line -6 of Theorem 8.3.2, is the correct version of ψ_k and consequently the formula in line -6 of Theorem 8.3.2 should have ψ_k and not $\bar{\psi}_k$ in it. With these remarks our two formulas completely agree.

2 A comment on Page 10

The discussion in paragraphs 3 and 4 of page 10 concerning the k -roots are not necessary for the definition of $\tilde{\alpha}$ and are incorrect in part. While the pairing $\langle \tilde{\beta}, \tilde{\gamma} \rangle$ with $\tilde{\beta} = \beta \in X(A_0) \otimes \mathbb{R}$ does not depend on the choice of $\tilde{\gamma}$, restricting to $\gamma \in X(A_0) \otimes \mathbb{R}$, $\langle \tilde{\gamma}, \tilde{\gamma} \rangle$ is not equal to $\langle \gamma, \gamma \rangle$ when G is not split as can be observed, for example, for $G = U(2, 1)$, where $(\gamma, \gamma) = 1/2$, $\gamma = \frac{1}{2}(\alpha + \beta)$, while $(\tilde{\gamma}, \tilde{\gamma}) = (\alpha, \alpha) = (\beta, \beta) = 2$, with α and β simple roots of $GL(3)$, restricting to $\frac{1}{2}(\alpha + \beta)$, the simple root of $U(2, 1)$. It must therefore be emphasized that the definition of $\tilde{\alpha}$ in (1.2.3) is exclusively given by (1.2.4) as explained in paragraph 2 of page 10 and the earlier literature on the method referred to as [Sh7] and [Sh8] in page 10.

3 A Comment on Pages 17 and 18

The calculations in pages 17 and 18 to compute the maximally split torus T of $U(2, 1)$ and its subtori T_0 and A using the Γ -stable \mathbb{Z} -lattice $\langle \alpha, \beta, \chi \rangle$, while giving them correctly,

is not the standard way to do so. In fact, to benefit from the results explained under 1.6.A, pages 15 and 16, one should use the full character lattice $X = \mathbb{Z}e_1 \oplus \mathbb{Z}e_2 \oplus \mathbb{Z}e_3$, where e_i , $i = 1, 2, 3$, are the coordinate functions $e_i(\text{diag}(t_1, t_2, t_3)) = t_i$, $i = 1, 2, 3$. (The \mathbb{Z} -lattice $\langle \alpha, \beta, \chi \rangle$ is a Γ -stable sublattice of X .) Then for $\chi \in X$, there are integers n_i , $i = 1, 2, 3$, such that $\chi = \sum_{i=1}^3 n_i e_i$. Moreover, using $\sigma(t) = w\bar{t}^{-1}w^{-1}$, we have that $\chi^\sigma = -\sum_{i=1}^3 n_i e_{4-i}$. Consequently, $X^\Gamma = \mathbb{Z}(e_1 - e_3)$ and $X_0 = \mathbb{Z}(e_1 + e_3) + \mathbb{Z}e_2$. The tori T_0 and A and their k -points are still exactly as in page 18. The reason for this agreement is that as a subgroup of T , $\langle \alpha, \beta, \chi \rangle^\perp = \langle \xi_3 \rangle$, the group of 3rd roots of 1, while $T^\perp = \{1\}$. The tori T and $T/\langle \xi_3 \rangle$ are isogeneous and thus, being tori, are isomorphic. Similarly for the subtori T_0 which contains $\langle \xi_3 \rangle$. The torus A is the same since $\xi_3 \notin A$. (There is a typo in the last line of page 17 : 2χ must be χ since $\chi + \chi^\sigma = 0$.)

4 Typos

This section consists of mathematical misprints or typos.

Page 7, Line +10: Change $|\varpi_v|$ to $|\varpi_v|_v$.

Page 8, Line +4: Change $\tilde{\phi}^+$ to $\tilde{\Phi}^+$.

Page 14, Line +16: Change $\sigma \in \mathfrak{p}(G)_k$ to $\sigma \in \mathfrak{p}(G)$.

Page 16, Line -13: Change $T = K^* \times K^*$ to $T(K) = K^* \times K^*$.

Page 18, Line +9: Change E^1 to K^1 .

Page 26, Line +8: Change $X_*(T)$ to $X_*(T)_k$.

Page 26, Line +14: Change $H_T(T)$ to $H_T(T(k))$.

Page 27, Line +21: Change $\mathcal{H}(\pi)^K$ to $\mathcal{H}(\sigma)^K$.

Page 32, Line -14: Remove $P_0 = M_0 N_0$.

Page 47, Line +13: Change *function* to *functional*.

Page 52, Line -7: Change *modulo* to *module*.

Page 52, Line +7: Change $\chi(w_0 n^- w_0^{-1})$ to $\bar{\chi}(w_0 n^- w_0^{-1})$.

Page 53, Line +7: Change $\varphi(nmg^*)$ to $f(nmg^*)$.

Page 53, Line +12: Add *of $U(k)$ -modules* after the exact sequence.

Page 53, Line +16: Change $I(\nu_0, \sigma)$ to $I(\nu, \sigma)$.

Page 53, Line -4: Change $x_0^{-1} P x_0 \cap N$ to $x_0^{-1} P x_0 \cap U$.

Page 54, Line -1: Change $\Lambda_n \in \mathfrak{a}_{\mathbb{C}}^*$ to $\Lambda_n \subseteq \mathfrak{a}_{\mathbb{C}}^*$.

- Page 57, Line +16: Change $U_M(k)$ to $U_{M'}(k)$, which is defined on Line -11.
- Page 58, Line +9: Change $u'_2 \in U_M(k)$ to $u'_2 \in U_{M'}(k)$.
- Page 58, Line +14: Change $\chi(x_0 u'_2 n_0^{-1})$ to $\chi(x_0 u'_2 x_0^{-1})$.
- Page 59, Line +9: Change *in* to *is*.
- Page 62, Line +6: Change $\begin{pmatrix} 1 & 1 & y \\ & 1 & -\bar{x} \\ & & 1 \end{pmatrix}$ to $\begin{pmatrix} 1 & x & y \\ & 1 & -\bar{x} \\ & & 1 \end{pmatrix}$.
- Page 73, Line -7: Change **direct product** to **semidirect product**.
- Page 73, Line -5: Change **ker** χ to $\cap_\chi \ker |\chi|$.
- Page 75, Line -1: Change $T(k)$ to $T(\mathbb{R})$.
- Page 76, Line +14: Change $G(\mathbb{R})$ -isomorphism to M_0 -isomorphism.
- Page 78, Line -10: Change **supp** φ to **supp** ψ .
- Page 80, Line +6: Change x to G .
- Page 79, Line -4: Change $x \in S$ to $x \in X$.
- Page 82, Line +9: Change $N_{\theta'} = N'$ to $N' = N_{\theta'}$.
- Page 90, Line +1: Replace “as discussed in Section 2.4” with “and if $V_i := \{X_{\beta^\vee} \in \mathfrak{h} | \langle \tilde{\alpha}, \beta^\vee \rangle = i\}$, then $r_i := r|_{V_i}$, $1 \leq i \leq m$.”
- Page 90, Line -8: Change unique root of α to unique root of B .
- Page 90, Line -8: Change $PGL_2(\mathbb{C})$ to \hat{A}_0 .
- Page 91, Line -5: Change $d^* \mu$ to $d^* x$.
- Page 100, Line -6: Change $n - 1$ to n .
- Page 101, Line +6: Change $1 \leq i \leq n - 1$ to $2 \leq i \leq n - 1$.
- Page 101, Line -16: In Equation (5.1.4), change $L(1 - s, \pi_n \times \pi_m)$ to $L(1 - s, \tilde{\pi}_n \times \tilde{\pi}_m)$.
- Page 102, Line -11: Change $\pi = \pi_n \otimes \tilde{\pi}_m \hookrightarrow \Pi_1$ to $\pi = \pi_n \otimes \tilde{\pi}_m \hookrightarrow \text{Ind } \Pi_1$.
- Page 109, Line +9: Change P_ρ to ρ_P .
- Page 112, Line -18: After the paragraph, add the sentence **Finally one needs to show that $\alpha + \alpha' \in \Phi_\theta^2$ whenever $\alpha, \alpha' \in \Phi_\theta^2$.**
- Page 113, Line -6: Change *Dedekind* to *Kronecker*.
- Page 116, Line +17: Change **the function** $f_{\varphi, v}$ to **the function** f_φ .
- Page 116, Line -5: Change $m_v = u_v^0 t_v k_v^0$ to $m_v = u_v^0 t_v k_v^0$.
- Page 116, Line -4: Change $H_P | M_0(k)$ to $H_P | M_0(\mathbb{Q}_v)$.
- Page 125, Line +8: Change $L^S(1 + is, \pi_v, \tilde{r}_{i, v})^{-1}$ to $L(1 + is, \pi_v, \tilde{r}_{i, v})^{-1}$.

- Page 125, Line +9:** Change $V(s\tilde{\alpha}, \pi_v)$ to $H(\pi_v)$.
- Page 125, Line -1:** Change $f_v(\varphi_v)$ to $f_v(e_v)$.
- Page 128, Line -1:** Add **then**, at the beginning of the line.
- Page 130, Line 10:** Change $f_S = \otimes_v f_{\varphi, s\tilde{\alpha}, v} = \otimes_v f_{\varphi_v, s\tilde{\alpha}}$ to $f_S = \otimes_{v \in S} f_{\varphi, s\tilde{\alpha}, v} = \otimes_{v \in S} f_{\varphi_v, s\tilde{\alpha}}$.
- Page 141, last line:** Change s_0 to $-is_0$.
- Page 143, Line -13:** Add **supercuspidal** after χ -generic.
- Page 169, Line +5:** Change $\omega_\pi = \otimes_v \omega_{\pi_v}$ **be the central character of π** to $\omega = \otimes_v \omega_v$ **be the central character of Π** .
- Page 169, Line +7:** Change ω_π to ω .
- Page 169, Line 11:** Change $v \in S, \pi_v$ to $v \notin S, \Pi_v$.
- Page 205, Line -15:** In reference [ShSp], change SL_6^* to SO_6^* .

5 Typesetting issues

These are minor typesetting issues and are unlikely to derail the reader.

- Page 76, Line -2:** Change $>>$ to \gg .
- Page 77, Line -15,-9:** Change $>>$ to \gg .
- Page 85, Line +12:** Change the bottom left entry in the matrix from \mathcal{O}_k to \mathcal{O}_k .
- Page 91, Line +11:** Change $(f_0 \begin{pmatrix} 1 & 0 \\ x & 1 \end{pmatrix})dx$ to $f_0(\begin{pmatrix} 1 & 0 \\ x & 1 \end{pmatrix})dx$.
- Page 115, Line -8:** Change $>$ to \rangle .
- Page 130, Line 11:** Change $<<$ to \ll .
- Page 199:** Bibliography should be alphabetical, for example, references [CKPSS1], [CKPSS2] should precede [CPS1], [CPS2] etc.

References

- [Sha10] Freydoon Shahidi, *Eisenstein series and automorphic L-functions*, American Mathematical Society Colloquium Publications, vol. 58, American Mathematical Society, Providence, RI, 2010. MR2683009
- [AGIKMS24] H. Atobe, W.T. Gan, A. Ichino, T. Kaletha, A. Minguez, and S.W. Shin, *Local intertwining relations and co-tempered A-packets of classical groups* (2024). arXiv: 2410.13504.