

Name: \_\_\_\_\_ PUID \_\_\_\_\_

**Instructions:**

- ◆ You must use a #2 pencil on the answer sheet.
- ◆ On the answer sheet, fill in: (This has to be correct for your score to load into Blackboard.)
  - a) Your last name, first name and middle initial and blacken the appropriate spaces.
  - b) Your section number and blacken the appropriate spaces.
  - c) Your 10-digit student id, (starting with 00...) and blacken the appropriate spaces.
  - d) The number 0X in the test/quiz number and blacken the 0 and the X.**
- ◆ Make sure that the cover of this exam matches the color of your answer sheet. If you are color blind, ask the person sitting next to you for assistance.
- ◆ There are 30 questions. On the answer sheet, blacken your choice of the correct answer for questions 1-30. Turn in BOTH the answer sheet and the exam. Only the answer sheet will be graded. The exam is collected as back up, so circle your answers. Do not leave without turning in your answer sheet and your exam.
- ◆ All questions are worth the same. Please answer every question!
- ◆ A TI-30 XA, one-line calculator may be used. No other calculator is allowed. All other electronics devices must be turned off and out of sight.
- ◆ No books or papers are allowed. You cannot bring in a unit circle or a formula sheet.
- ◆ The exam is self-explanatory. Do not ask questions about any of the exam problems unless you feel there is a typo.
- ◆ Exam answers will not be posted after the exam. Exam scores should be posted in the grade book in Blackboard by Thursday, December 18<sup>th</sup>. If you feel there is an error, contact Tim Delworth, delworth@ purdue.edu.

The Formula Sheet is the next page of the exam. You are welcome to detach it from the exam if that is what your kids are into these days. In my day, we were just happy to ...

## MA 154 FORMULA SHEET

## ADDITION AND SUBTRACTION FORMULAS

$$\sin(u + v) = \sin u \cos v + \cos u \sin v$$

$$\sin(u - v) = \sin u \cos v - \cos u \sin v$$

$$\cos(u + v) = \cos u \cos v - \sin u \sin v$$

$$\cos(u - v) = \cos u \cos v + \sin u \sin v$$

$$\tan(u + v) = \frac{\tan u + \tan v}{1 - \tan u \tan v}$$

$$\tan(u - v) = \frac{\tan u - \tan v}{1 + \tan u \tan v}$$

## DOUBLE-ANGLE FORMULAS

$$\sin(2u) = 2 \sin u \cos u$$

$$\cos(2u) = \cos^2 u - \sin^2 u$$

$$\tan(2u) = \frac{2 \tan u}{1 - \tan^2 u}$$

## PYTHAGOREAN IDENTITIES

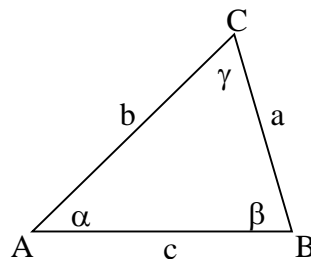
$$\sin^2 \theta + \cos^2 \theta = 1$$

$$1 + \tan^2 \theta = \sec^2 \theta$$

$$1 + \cot^2 \theta = \csc^2 \theta$$

## LAW OF SINES

$$\frac{\sin \alpha}{a} = \frac{\sin \beta}{b} = \frac{\sin \gamma}{c}$$



## LAW OF COSINES

$$c^2 = a^2 + b^2 - 2ab \cos \gamma$$

## ANGLE BETWEEN TWO VECTORS:

$$\cos \theta = \frac{(\vec{a}) \cdot (\vec{b})}{\|\vec{a}\| \|\vec{b}\|}$$

## QUADRATIC FORMULA:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

## STANDARD FORM FOR CONICS:

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$

$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$$

$$(y-k)^2 = 4p(x-h)$$

$$\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1$$

$$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$$

$$(x-h)^2 = 4p(y-k)$$

1. Find the angle between the vectors  $\langle -8, 9 \rangle$  and  $\langle 6, 1 \rangle$  to the nearest minute.

- A.  $115^\circ 31'$
- B.  $57^\circ 50'$
- C.  $122^\circ 10'$
- D.  $64^\circ 29'$
- E. None of the above

2. Determine  $m$  such that the two vectors are orthogonal.

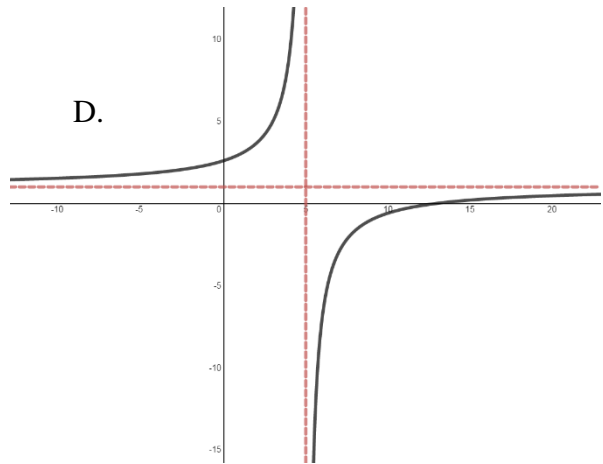
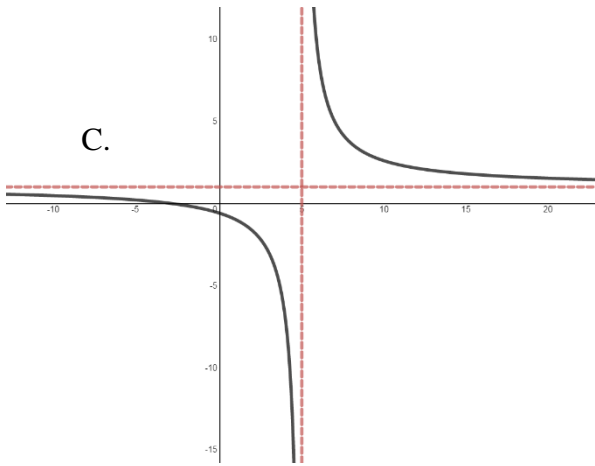
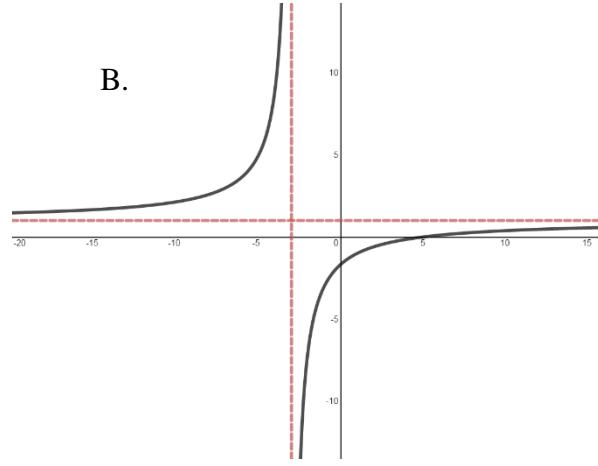
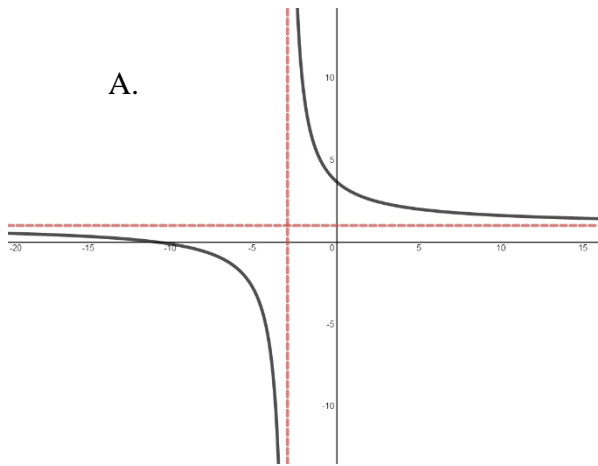
$$a = 4mi + 6j \qquad b = 9i - 3j$$

- A.  $m = \frac{1}{2}$
- B.  $m = \frac{9}{8}$
- C.  $m = \frac{-1}{2}$
- D.  $m = \frac{-9}{8}$
- E. None of the above

3. What is the **range** of the function,  $f(x) = \frac{6}{x}$ ?

- A.  $R = (0, \infty)$
- B.  $R =$  all real numbers
- C.  $R = (-\infty, 0)$
- D.  $R = (-\infty, 0) \cup (0, \infty)$
- E. None of the above.

4. Which of the following graphs best represents the function,  $f(x) = \frac{x-5}{x+3}$  ?



5. Find the equation of the vertical asymptote(s) of the function  $f(x) = \frac{3x^2 - 7x + 2}{x^2 + 2x - 8}$ .

- A.  $x = -4$
- B.  $x = 3, x = 2$
- C.  $x = 2$
- D.  $x = -4, x = 2$
- E. None of the above

6. Find the equation of the horizontal asymptote of the function  $f(x) = \frac{2x^2 - 4x + 2}{x^3 - 9x}$ .

- A.  $x = 4$
- B.  $y = 2$
- C.  $y = 1$
- D.  $y = 0$
- E. None of the above

7. Find an equation of a rational function  $f$  that satisfies the given conditions.

vertical asymptotes:  $x = -1, x = 2$

horizontal asymptote:  $y = -5$

x-intercepts:  $-3, 1$

A.  $f(x) = \frac{-5(x+3)(x-1)}{(x+1)(x-2)}$

B.  $f(x) = \frac{-5(x+1)(x-2)}{(x+3)(x-1)}$

C.  $f(x) = \frac{-5(x+3)(x-2)}{(x+1)(x-1)}$

D.  $f(x) = \frac{-5(x+1)(x-1)}{(x+3)(x-2)}$

- E. None of the above

8. Find an equation of the parabola that satisfies the given conditions.

Vertex  $V(4, 2)$ , Focus  $F(4, 7)$

A.  $(x+4)^2 = 5(y+2)$

B.  $(y-2)^2 = 20(x-4)$

C.  $(x-4)^2 = 5(y-2)$

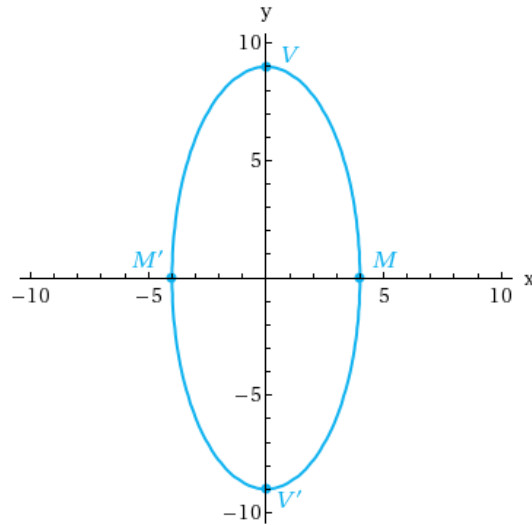
D.  $(y+2)^2 = 20(x+4)$

- E. None of the above

9. Which of the following best describes the parabola  $(y-7)^2 = -8(x+4)$ ?
- A. Its vertex is in QIV and it opens down.
  - B. Its vertex is in QII and it opens to the left.
  - C. Its vertex is in QII and it opens down.
  - D. Its vertex is in QIV and it opens to the left.
  - E. None of the above are true.

10. Find an equation for the ellipse shown.

- A.  $\frac{x^2}{81} + \frac{y^2}{16} = 1$
- B.  $\frac{x^2}{9} + \frac{y^2}{4} = 1$
- C.  $\frac{x^2}{4} + \frac{y^2}{9} = 1$
- D.  $\frac{x^2}{16} + \frac{y^2}{81} = 1$



11. Find the vertices of the ellipse with the equation  $\frac{(x-4)^2}{25} + \frac{(y-2)^2}{9} = 1$
- A.  $V(1,2), V'(7,2)$
  - B.  $V(-1,2), V'(9,2)$
  - C.  $V(4,5), V'(4,-1)$
  - D.  $V(4,-3), V'(4,7)$
  - E. None of the above

12. Find an equation for the ellipse that has its center at the origin vertices  $V(\pm 6, 0)$ , passing through  $(2, 1)$

A.  $\frac{x^2}{36} + \frac{9y^2}{8} = 1$

B.  $\frac{35x^2}{144} + \frac{y^2}{36} = 1$

C.  $\frac{x^2}{36} + \frac{8y^2}{9} = 1$

D.  $\frac{144x^2}{35} + \frac{y^2}{36} = 1$

13. Assume that the length of the major axis of a planet's orbit is 184,000,000 miles and its eccentricity is 0.021. Approximate the **maximum** distance between the planet and the sun.

**Kepler's 1<sup>st</sup> Law of Planetary Motion:** "The orbit of a planet is an ellipse with the Sun at one of the two foci."

- A. 93,932,000 *miles*
- B. 180,136,000 *miles*
- C. 91,475,000 *miles*
- D. 187,846,000 *miles*
- E. 90,068,000 *miles*
14. Identify the graph of the equation as a parabola (with vertical or horizontal axis), circle, ellipse, or hyperbola.

$$x^2 + 2x = 2y - 4$$

- A. hyperbola
- B. circle
- C. parabola with horizontal axis
- D. ellipse
- E. parabola with vertical axis

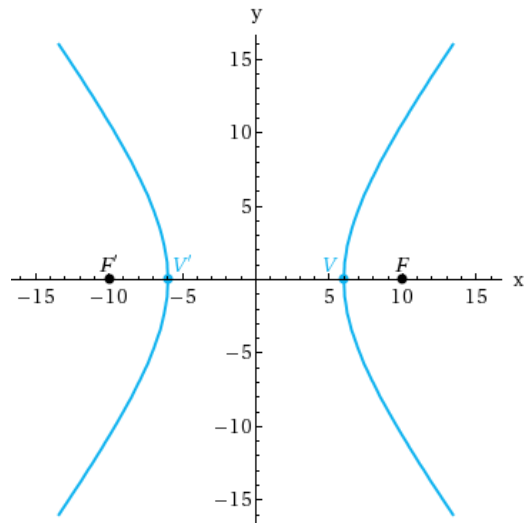
15. Find an equation for the hyperbola shown.

A.  $\frac{x^2}{64} - \frac{y^2}{36} = 1$

B.  $\frac{y^2}{36} - \frac{x^2}{64} = 1$

C.  $\frac{x^2}{36} - \frac{y^2}{64} = 1$

D.  $\frac{y^2}{64} - \frac{x^2}{36} = 1$



16. Find the exact value of  $\sin \theta$  if  $\theta$  is in standard position and the terminal side of  $\theta$  is in  $QII$  and parallel to the line  $10x + 5y = 15$

A.  $\sin \theta = \frac{-1}{\sqrt{5}}$

B.  $\sin \theta = \frac{2}{\sqrt{5}}$

C.  $\sin \theta = \frac{1}{\sqrt{5}}$

D.  $\sin \theta = \frac{-2}{\sqrt{5}}$

E. None of the above

17. Approximate, to the nearest 0.01 **radians**, all angles  $\theta$  in the interval  $[0, 2\pi)$  that satisfy equation  $\tan \theta = -0.8765$  (Hey, are those radians?)

A.  $\theta = 2.42, 3.86$

B.  $\theta = 2.29, 5.43$

C.  $\theta = 2.42, 5.56$

D.  $\theta = 2.29, 3.99$

E. None of the above

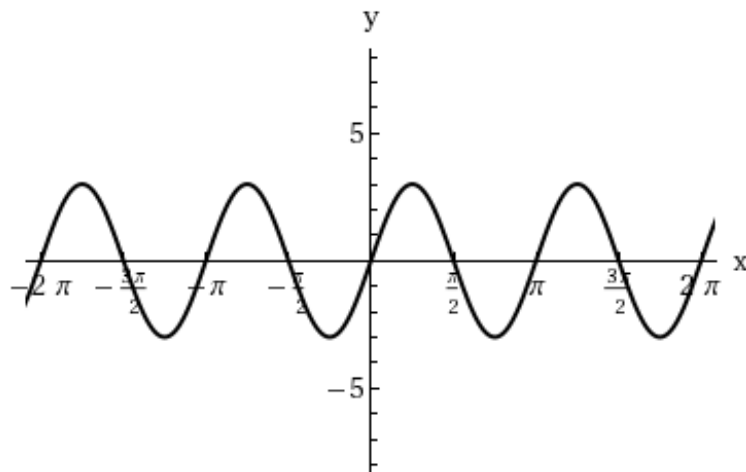


18. Approximate, to the nearest  $0.1^\circ$ , all angles  $\theta$  in the interval  $[0^\circ, 360^\circ)$  that satisfy equation  $\csc \theta = 2.3456$ . (Hey, did we just switch back to degrees?)

- A.  $\theta = 25.2^\circ, 154.8^\circ$
- B.  $\theta = 64.8^\circ, 115.2^\circ$
- C.  $\theta = 25.2^\circ, 334.8^\circ$
- D.  $\theta = 64.8^\circ, 295.2^\circ$
- E. None of the above

19. Find the Period of the given graph of the sine equation.

- A. Period = 3
- B. Period =  $2\pi$
- C. Period =  $\frac{\pi}{2}$
- D. Period = 6
- E. Period =  $\pi$



20. Express  $\theta = 2.2$  in terms of degrees, minutes, and seconds, to the nearest second.

- A.  $126^\circ 51' 47''$
- B.  $126^\circ 3' 3''$
- C.  $126^\circ 53' 52''$
- D.  $126^\circ 5' 7''$
- E. None of the above

Questions 21 and 22. The following is a love story of two ships that do not pass in the night.

A ship leave port at 1:00 pm and sails in the direction  $N 35^\circ E$  at a rate of 50 mph. Another ship leaves the same port at the same time and sails in the direction  $N 55^\circ W$  at a rate of 20 mph.

21. To the nearest whole mile, approximately how far apart are the ships at 6:00 pm?

- A. 269 miles
- B. 323 miles
- C. 215 miles
- D. 303 miles
- E. 254 miles

22. What is the bearing, to the nearest degree, from the first ship to the second?

- A.  $S63^\circ W$
- B.  $S61^\circ W$
- C.  $S59^\circ W$
- D.  $S57^\circ W$
- E.  $S55^\circ W$

23. Find all solutions of the equation in the interval  $[0, 2\pi)$ .

$$\sin t + \sin 2t = 0$$

A.  $t = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{2\pi}{3}, \frac{4\pi}{3}$

B.  $t = 0, \pi, \frac{2\pi}{3}, \frac{4\pi}{3}$

C.  $t = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{\pi}{3}, \frac{5\pi}{3}$

D.  $t = \frac{\pi}{4}, \frac{2\pi}{3}, \frac{5\pi}{4}, \frac{4\pi}{3}$

E.  $t = 0, \pi, \frac{\pi}{3}, \frac{5\pi}{3}$

24. Find all solutions of the equation using  $n$  as an arbitrary integer.

$$\sin \theta = 1$$

A.  $\theta = 2\pi n$

B.  $\theta = \frac{\pi}{2} + 2\pi n$

C.  $\theta = \frac{\pi}{4} + \pi n$

D.  $\theta = \frac{\pi}{2} + \pi n$

E.  $\theta = \pi n$

25. Express as a trigonometric function of one angle.

$$\cos(35^\circ)\cos(15^\circ) - \sin(35^\circ)\sin(15^\circ)$$

A.  $\sin(20^\circ)$

B.  $\cos(20^\circ)$

C.  $\sin(50^\circ)$

D.  $\cos(50^\circ)$

E. None of the above

26. The vectors  $a$  and  $b$  represent two forces acting at the same point, and  $\theta$  is the smallest positive angle between  $a$  and  $b$ . Approximate the magnitude of the resultant force to one decimal place.

$$\|a\| = 7lb, \quad \|b\| = 12lb, \quad \theta = 100^\circ$$

- A.  $\|r\| = 11.0 lb$
- B.  $\|r\| = 12.8 lb$
- C.  $\|r\| = 14.7 lb$
- D.  $\|r\| = 17.3 lb$
- E. None of the above
27. Find a vector that has the same direction as  $24i - 7j$  and 6 times the magnitude.
- A.  $-144i + 42j$
- B.  $\frac{-144}{25}i + \frac{42}{25}j$
- C.  $144i - 42j$
- D.  $\frac{144}{25}i - \frac{42}{25}j$
- E. None of the above
28. Find the perimeter of  $\triangle ABC$ , given angle  $\gamma = 100^\circ$ , side  $b = 260$ , and side  $c = 350$ . Round to the nearest whole number.
- A. 803
- B. 815
- C. 789
- D. 797
- E. None of the above

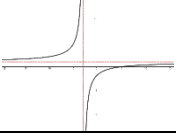
Questions 29 and 30: An airplane with an airspeed of 450 mph is flying in the direction  $140^\circ$  and a 75 mph wind is blowing in the direction of  $60^\circ$ .

29. What is the ground speed of the plane? Round to the nearest whole number.

- A.  $458\text{mph}$
- B.  $482\text{mph}$
- C.  $475\text{mph}$
- D.  $469\text{mph}$
- E. None of the above

30. What is the true course of the plane? Round to the nearest whole degree.

- A.  $135^\circ$
- B.  $142^\circ$
- C.  $131^\circ$
- D.  $144^\circ$
- E. None of the above

Question	Answer	Letter	Question	Answer	Letter
1.	$122^\circ 10'$	C	16.	$\sin \theta = \frac{2}{\sqrt{5}}$	B
2.	$m = \frac{1}{2}$	A	17.	$\theta = 2.42, 5.56$	C
3.	$R = (-\infty, 0) \cup (0, -\infty)$	D	18.	$\theta = 25.2^\circ, 154.8^\circ$	A
4.		B	19.	Period = $\pi$	E
5.	$x = -4$	A	20.	$126^\circ 3' 3''$	B
6.	$y = 0$	D	21.	269 miles	A
7.	$f(x) = \frac{-5(x+3)(x-1)}{(x+1)(x-2)}$	A	22.	$S57^\circ W$	D
8.	$(x-4)^2 = 20(y-2)$	E	23.	$t = 0, \pi, \frac{2\pi}{3}, \frac{4\pi}{3}$	B
9.	Its vertex is in QII and it opens to the left.	B	24.	$\theta = \frac{\pi}{2} + 2\pi n$	B
10.	$\frac{x^2}{16} + \frac{y^2}{81} = 1$	D	25.	$\cos(50^\circ)$	D
11.	$V(-1, 2), V'(9, 2)$	B	26.	$\ r\  = 12.8 \text{ lb}$	B
12.	$\frac{x^2}{36} + \frac{8y^2}{9} = 1$	C	27.	$144i - 48j$	C
13.	93,932,000 miles	A	28.	803	A
14.	parabola with vertical axis	E	29.	469 mph	D
15.	$\frac{x^2}{36} - \frac{y^2}{64} = 1$	C	30.	$131^\circ$	C