MA 16100 - Exam 1 - 09/21/2023

NAME ___________________ YOUR TA’S NAME ___________________

STUDENT ID # ___________ RECITATION TIME ___________

Be sure the paper you are looking at right now is GREEN and matched with the color of the scantron! Write [11] in the TEST/QUIZ NUMBER boxes and blacken the appropriate spaces on the scantron. Use a #2 pencil for the scantron and fill in:

1. Your name. If there’s not enough space, fill in as much as you can.
2. Section number. If you don’t know your section number, ask your TA.
3. Test/Quiz number: [11]
4. Student Identification Number: [Your Purdue ID Number with two leading zeros]

There are 12 questions, each worth 8 points (you will automatically earn 4 points for filling out your student ID number correctly). Blacken your answer choice on the scantron for questions 1-12. Use this exam booklet for all your work and use the back of the test pages for scrap paper. Submit both the scantron and the exam booklet when finished.

If you finish the exam before 8:50 PM, you may leave the room after turning in the scantron sheet and the exam booklet. You may not leave the room before 8:20 PM. If you don’t finish by 8:50 PM, you MUST REMAIN SEATED until your TA collects your materials.

EXAM POLICIES

1. Students may not open the exam until instructed to do so.
2. Students must obey the orders and requests by all proctors, TAs, and lecturers.
3. No student may leave in the first 20 min or in the last 10 min of the exam.
4. Books, notes, calculators, or any electronic devices are not allowed on the exam, and they should not even be in sight in the exam room. Students may not look at anybody else’s test, and may not communicate with anybody else except, if they have a question, with their TA or lecturer.
5. After time is called, the students have to put down all writing instruments and remain in their seats, while the TAs will collect the scantrons and the exams.
6. Any violation of these rules and any act of academic dishonesty may result in severe penalties. Additionally, all violators will be reported to the Office of the Dean of Students.

I have read and understand the exam rules stated above:

STUDENT SIGNATURE: ________________________________
1. The Domain of the function \( f(x) = \sqrt{2-x} + \frac{1}{\sqrt{x^2 - 9}} \) is

A. \([2, 3)\)
B. \((-\infty, \infty)\)
C. \((-\infty, -3)\)
D. \((-\infty, -3) \cup (3, \infty)\)
E. \((-\infty, 2] \cup (3, \infty)\)
2. Given the graph of \( y = f(x) \), how do you obtain the graph of \( y = f(3x + 6) + 8 \)?

A. Shift the graph to the right by 2 units, stretch the graph horizontally by a factor of 3. Then shift the graph up by 8 units.

B. Shift the graph to the left by 6 units, compress the graph horizontally by a factor of 3. Then shift the graph up by 8 units.

C. Shift the graph to the left by 2 units, compress the graph vertically by a factor of 3. Then shift the graph down by 8 units.

D. Shift the graph to the left by 2 units, compress the graph horizontally by a factor of 3. Then shift the graph up by 8 units.

E. Shift the graph to the left by 6 units, stretch the graph horizontally by a factor of 3. Then shift the graph up by 8 units.
3. Solve $7^{2x+5} = 35$.

A. $\frac{-4 + \log_7 5}{2}$
B. $\frac{5 + \log_7 5}{2}$
C. $\frac{-4 + \log_5 7}{2}$
D. $\frac{-6 + \log_7 5}{2}$
E. $\frac{4 + \log_7 5}{2}$
4. Determine the limit

\[ \lim_{x \to -2^+} \frac{x^2 - 2x - 8}{x + 2} \]

A. $-\infty$
B. $+\infty$
C. $-2$
D. 6
E. $-6$
5. Find all vertical and horizontal asymptotes of the function \( f(x) = \frac{(3x - 2)^2(x - 4)^3}{(2x - 1)^3(x^2 - 16)} \).

<table>
<thead>
<tr>
<th>Vertical Asymptotes</th>
<th>Horizontal Asymptotes</th>
</tr>
</thead>
<tbody>
<tr>
<td>A. ( x = \frac{1}{2}, x = 4 ) and ( y = \frac{9}{8} )</td>
<td></td>
</tr>
<tr>
<td>B. ( x = \frac{1}{2}, x = -4 ) and ( y = \frac{9}{8} )</td>
<td></td>
</tr>
<tr>
<td>C. ( x = \frac{1}{2}, x = 4, x = -4 ) and ( y = \frac{9}{8} )</td>
<td></td>
</tr>
<tr>
<td>D. ( x = \frac{1}{2}, x = -4 ) and ( y = \frac{3}{2} )</td>
<td></td>
</tr>
<tr>
<td>E. ( x = \frac{1}{2}, x = 4 ) and ( y = \frac{3}{2} )</td>
<td></td>
</tr>
</tbody>
</table>
6. Consider the function
\[ f(x) = \begin{cases} 
-2x + 5 & x < 2 \\
-(x - 3)^2 + 3 & x \geq 2.
\end{cases} \]

Which of the following statements are true?

(I) \( f(2) \) is defined
(II) \( \lim_{x \to 2^-} f(x) = 1 \)
(III) \( \lim_{x \to 2^+} f(x) = 1 \)
(IV) \( \lim_{x \to 2} f(x) \) exists

A. I and II only
B. I, II, and IV only
C. All of them
D. I and IV only
E. II, III, and IV only
7. Find the values of $a$ and $b$ so that $f(x)$ defined below is continuous for all values of $x$.

$$f(x) = \begin{cases} 
  x^2 + 2, & x < -1 \\
  ax + b, & -1 \leq x \leq 1 \\
  \frac{x^2 + 9}{x + 1}, & x > 1 
\end{cases}$$

A. $a = 0, b = 5$
B. $a = 0, b = 3$
C. $a = 1, b = 4$
D. $a = -1, b = 4$
E. No value of $a$ or $b$, since $\frac{x^2 + 9}{x + 1}$ is not continuous at $x = -1$
8. Evaluate:

\[
\lim_{x \to 0} \frac{x}{\sqrt{x^2 + 36} - 6}
\]

A. \(-\infty\)
B. \(\infty\)
C. 6
D. 12
E. The limit does not exist
9. The function

\[ f(x) = \begin{cases} 
8x + 5, & x < 1 \\
13, & x = 1 \\
-x + 13, & x > 1 
\end{cases} \]

is NOT continuous at \( x = 1 \) because:

A. \( f(1) \) is not defined
B. \( \lim_{x \to 1} f(x) \) does not exits
C. \( \lim_{x \to 1} f(x) \neq f(1) \)
D. There is a vertical asymptote at \( x = 1 \)
E. None of the above options, the function is actually continuous at \( x = 1 \)
10. Let $f(x) = 4x^2 - 2x + 8$. Use the definition of the derivative to find $f'(3)$. When you simplify the terms inside the limit, you get:

A. $f'(3) = \lim_{h \to 0} (16 + 4h)$
B. $f'(3) = \lim_{h \to 0} (16h + 4h^2)$
C. $f'(3) = \lim_{h \to 0} (22 + 6h)$
D. $f'(3) = \lim_{h \to 0} (22 + 4h)$
E. $f'(3) = \lim_{h \to 0} (24h + h^2 - 2)$
11. Let \( f(x) = \frac{5}{x} \). Find the equation of the line tangent to the graph of \( f(x) \) at the point \((4, f(4))\).

A. \( y = \frac{5}{4}(x - 4) + \frac{5}{4} \)
B. \( y = -\frac{5}{16}(x - 4) + \frac{5}{4} \)
C. \( y = \frac{5}{4}(x - 4) + \frac{5}{16} \)
D. \( y = -\frac{5}{16}(x - 4) - \frac{5}{16} \)
E. \( y = -\frac{5}{16}(x - 4) - \frac{5}{4} \)
12. Suppose \( f(x) = -3 + \sqrt{3x + 22} \), find \( f^{-1}(1) \).

A. \(-5\)
B. \(2\)
C. \(8\)
D. \(-2\)
E. \(-8\)