

Name: \_\_\_\_\_

10-digit PUID: \_\_\_\_\_

Lecturer: \_\_\_\_\_

Recitation Instructor: \_\_\_\_\_

Recitation Time: \_\_\_\_\_

## Instructions:

1. This package contains 12 problems worth 8 points each.
2. Please supply all information requested. On the scantron sheet, print your name, your division-section number and 10 digit PUID number in addition to filling in the corresponding circles. You get 4 points for supplying all information correctly.
3. Work only in the space provided, or on the backside of the pages. Circle your choice for each problem in this booklet, and mark your answer on the scantron sheet.
4. No books, notes, calculator or any electronic devices, please.

1 B    2 B    3 E    4 B    5 D    6 B  
7 B    8 D    9 B    10 A    11 C    12 E

1. A spherical snowball is melting evenly in such a way that its volume decreases at the rate of  $1 \text{ cm}^3$  per minute. At what rate is the radius decreasing when it is 3 cm long? (Recall the volume of the sphere is  $\frac{4\pi r^3}{3}$ , where  $r$  is the radius.)

A.  $18\pi \text{ cm/min}$

B.  $\frac{1}{36\pi} \text{ cm/min}$

C.  $36\pi \text{ cm/min}$

D.  $\frac{1}{9\pi} \text{ cm/min}$

E.  $9\pi \text{ cm/min}$

2. What approximate value do you get for  $\ln 1.1$  if you use linear approximation at 1?

A. 1.1

B. 0.1

C.  $1/11$

D.  $9/11$

E. 1

3. The maximum value of  $2x^3 - 3x^2 + 1$  over the interval  $[-1, 2]$  is

- A. 0
- B. 1
- C. 8
- D. 6
- E. 5

4. The minimum value of  $\sqrt{x + \frac{1}{16x^2}}$  for  $x > 0$  is

- A. 2
- B.  $\frac{\sqrt{3}}{2}$
- C.  $\frac{\sqrt{2}}{2}$
- D.  $1/2$
- E.  $\frac{\sqrt{3}}{4}$

5. Consider the function  $f(x) = x^5 - 3x + a$ . Which of the following statements are always true, regardless of what the constant  $a$  is?

- I.  $f$  has at least one root on  $[1, 2]$ ;
- II.  $f$  has at most one root on  $[1, 2]$ ;
- III.  $f$  is increasing on  $[1, 2]$ .

- A. Only I.
- B. Only II.
- C. Only III.
- D. Only II and III.
- E. All are true.

6. Supposing  $h(x)$  is a differentiable function defined for  $-\infty < x < \infty$ , and  $x_0$  is a real number, which is true?

- I. If  $h$  has a local minimum at  $x_0$ , then  $h(x_0) < 0$ .
- II. If  $h$  has a local minimum at  $x_0$ , then  $h'(x_0) = 0$ .
- III. If  $h'(x_0) = 0$ , then  $h$  has a local minimum or maximum at  $x_0$ .

- A. Only I.
- B. Only II.
- C. Only III.
- D. Only I and II.
- E. Only II and III.

7. The second derivative of a function  $\varphi$  is given by  $\varphi''(x) = (x - 1)^2x(x + 1)$ . Then  $\varphi$  is concave down on

- A.  $(-\infty, -1)$ .
- B.  $(-1, 0)$ .
- C.  $(0, 1)$ .
- D.  $(0, \infty)$ .
- E.  $(-\infty, -1)$  and  $(1, \infty)$ .

8. The function  $3x^5 - 5x^3$  has

- A. a local minimum at  $-1$  and a local maximum at  $1$ ;
- B. a local minimum at  $-1$  and  $1$ ;
- C. an inflection point at  $-1$  and a local minimum at  $0$ ;
- D. a local maximum at  $-1$  and a local minimum at  $1$ ;
- E. inflection points at  $-1, 1,$  and  $0$ .

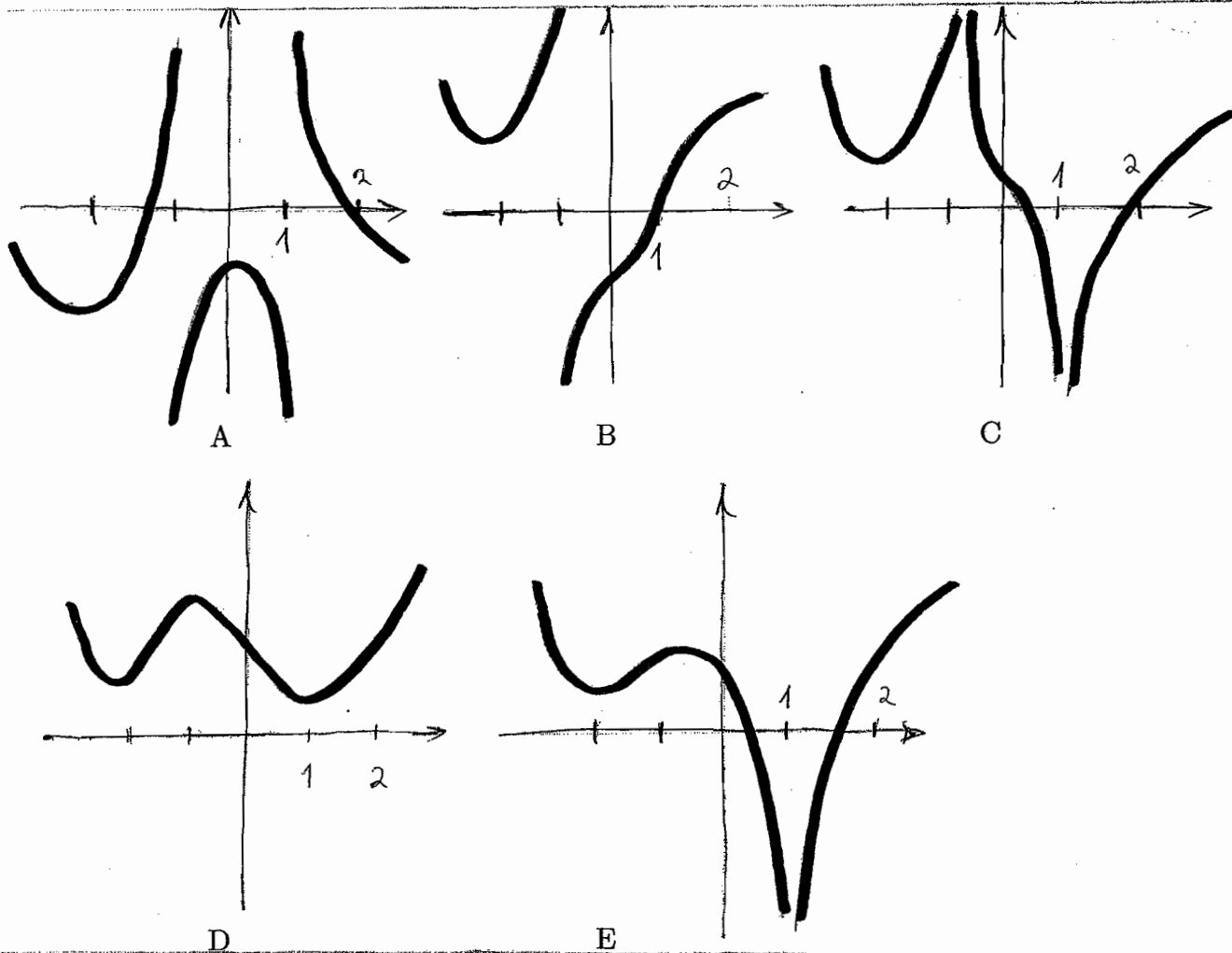
9.  $\lim_{x \rightarrow 0} \frac{x \sin x}{1 - \cos 2x} =$

- A.  $\frac{1}{4}$
- B.  $\frac{1}{2}$
- C. 0
- D. -1
- E.  $\infty$

10.  $\lim_{x \rightarrow \pi/2} \tan x \ln \sin x =$

- A. 0
- B. 1
- C. -1
- D.  $\ln \frac{\pi}{2}$
- E.  $-\infty$

11. Which could be the graph of the function  $\varphi$  if its derivative is  $\varphi'(x) = \frac{x+2}{x^2-1}$



12. If the sum of the length of the legs of a right triangle is 7, what is the minimum length of its hypotenuse?

- A. 8
- B. 7
- C. 5
- D.  $5\sqrt{2}$
- E.  $\frac{7\sqrt{2}}{2}$