MA161 - Fall 2009 - FINAL EXAM

(1) The domain of the function $f(x) = \sqrt{\ln(x+5)}$ is:

(a) $[0, \infty)$

(b) $[-5, \infty)$

(c) $[5, \infty)$

(d) $[-4, \infty)$

(e) $[4, \infty)$

(2) Let $f(x) = \sqrt[3]{2-x}$. Which of the following is $f^{-1}(x)$?

(a) $(2-x)^3$

(b) $(x-2)^3$

(c) $2 + x^3$

(d) $x^3 - 2$

(e) $2 - x^3$

(3) If $f(x) = \begin{cases} x^2+9 & \text{for } x \leq 1 \\ 12x - ax^2 & \text{for } x > 1 \end{cases}$ determine all values of a so that $f(x)$ is continuous at all values of x.

(a) $a = 0$

(b) $a = 1$

(c) $a = 2$

(d) $a = 3$

(e) There are no such values of a

(4) If $f(x) = x^2 \tan x$, the slope of the tangent line at $(\frac{\pi}{3}, f(\frac{\pi}{3}))$ is:

(a) $\dfrac{4\pi^2 + 6\sqrt{3}\pi}{27}$

(b) $\dfrac{4\pi^2 + 6\sqrt{3}\pi}{9}$

(c) $\dfrac{2\sqrt{3}\pi^2 + 2\sqrt{3}\pi}{9}$

(d) $\dfrac{8\pi}{3}$

(e) $\dfrac{4\pi\sqrt{3}}{3}$

(5) If $f(x) = \dfrac{x^3 - 2x}{x^2 + 1}$, then $f'(2) =$

(a) 34/25

(b) 66/25

(c) 5/2

(d) 1/5

(e) −2/25

(6) If $f(x) = e^{4x}$, evaluate $\lim_{h \to 0} \dfrac{f(3+h) - f(3)}{h}$.

(a) e^{12}

(b) e^{7}

(c) $4e^{12}$

(d) $4e^{7}$

(e) ∞

(7) The function $f(x) = \dfrac{x^2+1}{x^3+8}$ has:

(a) no vertical or horizontal asymptotes

(b) 1 vertical and 1 horizontal asymptote

(c) 2 vertical and 1 horizontal asymptote

(d) 1 vertical and 2 horizontal asymptotes

(e) 1 vertical and no horizontal asymptotes

(8) A particle moves on a line with velocity $v(t) = t - \ln(t^2+1)$. What is its maximum velocity on the interval $0 \leq t \leq 2$?

(a) $1 - \ln 2$

(b) 0

(c) $2 - \ln 5$

(d) $\ln 2 - 1$

(e) $\ln 5 - 2$

(9) Assume that f and g are differentiable functions defined on $(-\infty, \infty)$, $f(0) = 6, f'(0) = 10, f(2) = 5, f'(2) = 4, g(0) = 2$, and $g'(0) = 3$. Let $h(x) = f(g(x))$. What is $h'(0)$?

(a) 4

(b) 8

(c) 10

(d) 12

(e) 30

(10) Assume that y is defined implicitly as a differentiable function of x by the equation $2x^3 + x^2y - xy^3 = 2$. Find $\frac{dy}{dx}$ at $(1, 1)$.

(a) $\frac{-3}{2}$

(b) $\frac{7}{2}$

(c) 0

(d) -3

(e) -4

(11) Evaluate $\lim_{x \to 0} \frac{\cos(2x) - 1}{x^2}$.

(a) -2

(b) -1

(c) 0

(d) 1

(e) 2

(12) Water is withdrawn at the constant rate of 2 ft^3 / min from a cone-shaped reservoir which has its vertex down. The diameter of the top of the tank measures 4 feet and the height of the tank is 8 feet . How fast is the water level falling when the depth of the water in the reservoir is 2 feet? (Recall that the volume of a cone of height h and radius r is $V = \frac{\pi}{3}r^2h$).

(a) $\frac{2}{\pi}$ ft/min

(b) $\frac{6}{\pi}$ ft/min

(c) $\frac{4}{\pi}$ ft/min

(d) $\frac{8}{\pi}$ ft/min

(e) $\frac{16}{\pi}$ ft/min

(13) At the beginning of an experiment a colony has N bacteria. Two hours later it has 4N bacteria. How many hours, measured from the beginning, does it take for the colony to have 10N bacteria?

(a) $\dfrac{\ln 5N}{\ln 2}$

(b) $\dfrac{N\ln 5}{2\ln 2}$

(c) $\dfrac{\ln 5}{\ln 2}$

(d) $4\dfrac{\ln N}{\ln 2}$

(e) $\dfrac{\ln 10}{\ln 2}$

(14) The approximate value of $(16.32)^{\frac{1}{4}}$ given by linear approximation is equal to

(a) 2.01

(b) 2.10

(c) 2.02

(d) 2.20

(e) 2.06

(15) Find the critical numbers of $f(x) = e^x \sin x$ for $0 \leq x \leq 2\pi$.

(a) $\pi/4$ and $5\pi/4$

(b) $3\pi/4$ and $7\pi/4$

(c) $\pi/4$ and $3\pi/4$

(d) $\pi/4$ and $7\pi/4$

(e) $\pi/4$ and $\pi/2$

(16) Compute $\int_1^4 (\sqrt{x} - \frac{1}{\sqrt{x}})\, dx$

(a) $2\sqrt{2} - 10/3$

(b) $\sqrt{2} - 1/3$

(c) $\sqrt{2} + 4/3$

(d) $2\sqrt{2} + 14/3$

(e) $8/3$

(17) Evaluate $\frac{d}{dx}\left(\int_0^{2x} \arctan t \, dt\right)$ at $x = \frac{1}{2}$.

(a) $\pi/3$

(b) 1

(c) $\pi/4$

(d) $\pi/2$

(e) 2

(18) A certain function $f(x)$ satisfies $f''(x) = 2 - 3x$. We also know that $f'(0) = -1$ and $f(0) = 1$. Compute $f(2)$.

(a) -1

(b) -3

(c) 3

(d) 1

(e) -2

(19) Compute $\lim_{x \to 0} (1-x)^{\frac{5}{x}}$.

(a) 1

(b) e^3

(c) e^{-5}

(d) e^{-3}

(e) e^5

(20) The derivative of a function f is given by $f'(x) = (x-1)^2(x-2)^3(x-3)$. Which of the following are correct?

I) $f(2)$ is a local maximum and $f(3)$ is a local minimum of $f(x)$.
II) $f(x)$ is increasing on the interval $(1,3)$.
III) $f(x)$ is decreasing on $(-\infty, 1)$ and increasing on $(1, \infty)$.

(a) only I is correct

(b) only I and III are correct

(c) only II is correct

(d) only II and III are correct

(e) only III is correct

(21) A rectangle is centered at the origin, its sides are parallel to the axes and all of its vertices lie on the curve $4x^2+y^2=8$. What is the maximum area of such rectangle?

(a) 4

(b) 8

(c) $4\sqrt{2}$

(d) $2\sqrt{2}$

(e) 2

(22) Compute $\displaystyle\int_0^1 \frac{3x^2}{\sqrt{x^3+1}}\,dx$

(a) $3\sqrt{2}-3$

(b) $2(\sqrt{3}-1)$

(c) 2

(d) $2(\sqrt{2}-1)$

(e) $6\sqrt{3}-4$

(23) On what intervals is the graph of $f(x) = x^4 + 4x^3 - 18x^2 - 6x$ concave downward?

(a) on $(-3, 1)$ and $(2, 3)$

(b) on $(-\infty, -3)$ and $(1, \infty)$

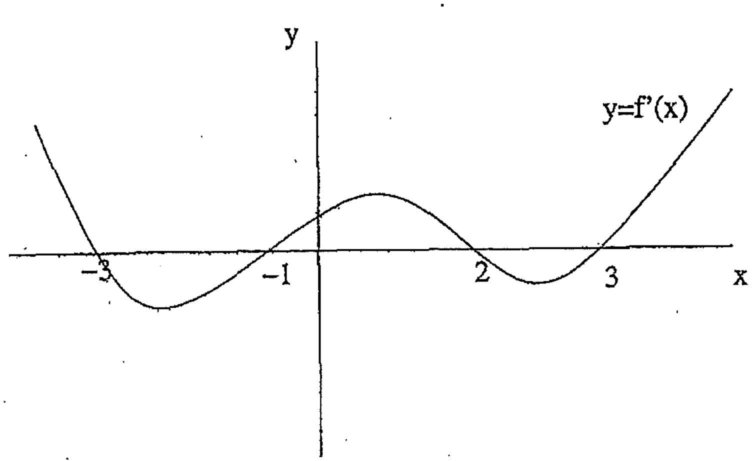
(c) only on $(-\infty, 11)$

(d) only on $(3, \infty)$

(e) on $(-3, 1)$

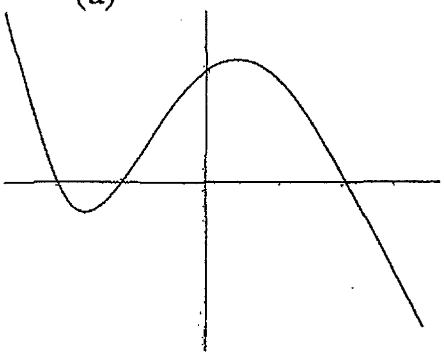
(24) The figure below illustrates the graph **of the derivative** of a differentiable function f which is defined in $(-4, 4)$. We can conclude that $f(x)$ achieves local maxima and minima at the following points:

(a) local maxima at -3 and 2 and local minima at -1 and 3
(b) local maxima at -1 and 3 and local minima at -3 and 2
(c) local maxima at -1 and 3 and local minimum at 2
(d) local maxima at -3 and 2 and local minimum at -1
(e) local maximum at a point between -3 and -1 and a local minimum at 0.



(25) The graph of the function $f(x) = -\frac{1}{3}x^3 - \frac{1}{2}x^2 + 2x + 2$ looks mostly like

(a)



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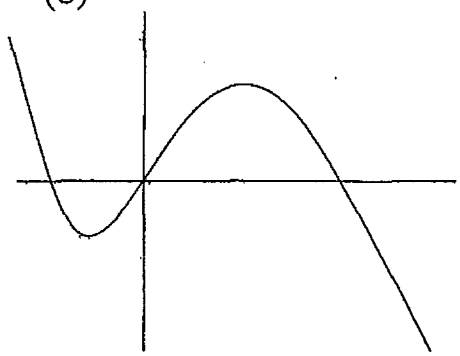

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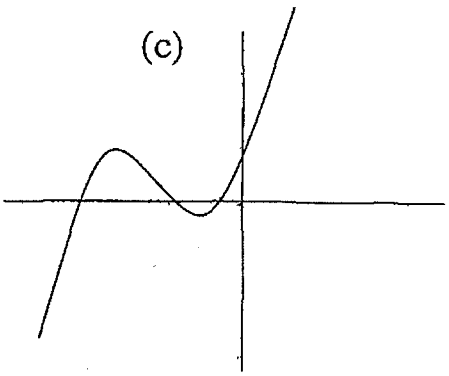

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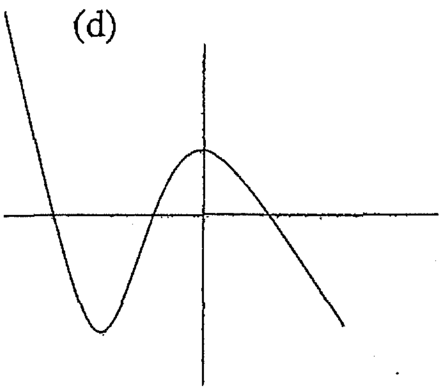

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