MA161 Final Exam, Fall 2013, December 9, 7:00-9:00pm

Name:____

10-digit PUID Number:______________________________________

Recitation Instructor:_______________________________________

Recitation Section Number and Time:______________________________

Mark TEST 01 on your scantron!

Instructions:

1. **Fill in all the information requested above and on the scantron sheet. On the scantron sheet fill in the little circles for your name, section number, and PUID.**
2. **This booklet contains 22 problems. The test booklet has five pages including this one.**
3. **For each problem mark your answer on the scantron sheet and also circle it in this booklet.**
4. **Work only on the pages of this booklet.**
5. **No books, notes, calculators, or any kind of electronic device are to be used during this test.**
6. **At the end of the exam turn in your exam and scantron sheet to your recitation instructor.**

1. Find the constants a and b such that

$$F(x) = \begin{cases} \dfrac{x^2 - x}{x - 1}, & x < 1 \\ ax^2 + bx + 1, & 1 \leq x < 2 \\ x + a, & x \geq 2 \end{cases}$$

is continuous for all x.

A. $a = 3,\ b = -1$

B. $a = 1,\ b = 1$

C. $a = -1,\ b = 3$

D. $a = 1,\ b = -1$

E. $a = 2,\ b = 0$

2. A sample of a radioactive element initially has mass of 24 gm. After 2 minutes the sample of that element has mass of 2 gm. When (in minutes) is the mass equal to 4 gm?

A. $\dfrac{2\ln 6}{\ln 12}$

B. $\dfrac{\ln 6}{\ln 12}$

C. $\dfrac{2\ln 12}{\ln 6}$

D. $\dfrac{3\ln 6}{2\ln 2}$

E. $\dfrac{3\ln 8}{\ln 3}$

3. Let $h(x) = f(g(x))$, where $g(1) = 2$, $g'(1) = 3$, $f(1) = 4$, $f'(1) = 5$, $f(2) = 6$, and $f'(2) = 7$. Then $h'(1) =$

   A. 5

   B. 7

   C. 15

   D. 21

   E. None of the above is correct

4. A rectangular cardboard box with no top has a rectangular base so that one side is twice as long as the other. If the box must have a volume of $\frac{4}{3}$ m^3, what should the height of the box be to minimize the amount of cardboard used?

   A. $\left(\frac{2}{3}\right)^{\frac{1}{3}}$ m

   B. $\frac{2}{3}$ m

   C. $\frac{1}{\sqrt{2}}$ m

   D. $\frac{2\sqrt{2}}{3}$ m

   E. $\frac{2}{\sqrt[3]{3}}$ m

5. What is the domain of $f(x) = \sqrt{4x - x^2} + \ln(1 - x)$?

A. $[0, 1) \cup (1, 4]$

B. $[0, 1)$

C. $(1, 4]$

D. $[1, 4]$

E. $(0, 1)$

6. Evaluate $\lim_{x \to 0^+} \left(\frac{1}{x} - \frac{1}{e^x - 1} \right)$

A. ∞

B. 0

C. $\frac{1}{2}$

D. 2

E. $-\infty$

7. The slope of the line tangent to the graph of

$$y^2 - 2x^3 + xy^4 + 13 = 0$$

at $(2, 1)$ is

A. $\frac{12}{5}$

B. $\frac{18}{5}$

C. $-\frac{5}{18}$

D. $\frac{23}{10}$

E. -13

8. $\int x^3(1-2x^4)^{1/4}dx =$

A. $-\frac{5}{32}(1-2x^4)^{5/4} + C$

B. $-\frac{1}{8}(1-2x^4)^{5/4} + C$

C. $\frac{4}{5}(1-2x^4)^{5/4} + C$

D. $\frac{2}{5}(1-2x^4)^{5/4} + C$

E. $-\frac{1}{10}(1-2x^4)^{5/4} + C$

9. Suppose that
$$\lim_{x\to 2} f(x) = 0 \quad \text{and} \quad \lim_{x\to 2} g(x) = \infty.$$
What can be said about $\lim_{x\to 2} f(x)g(x)$?

A. $\lim_{x\to 2} f(x)g(x) = 0$

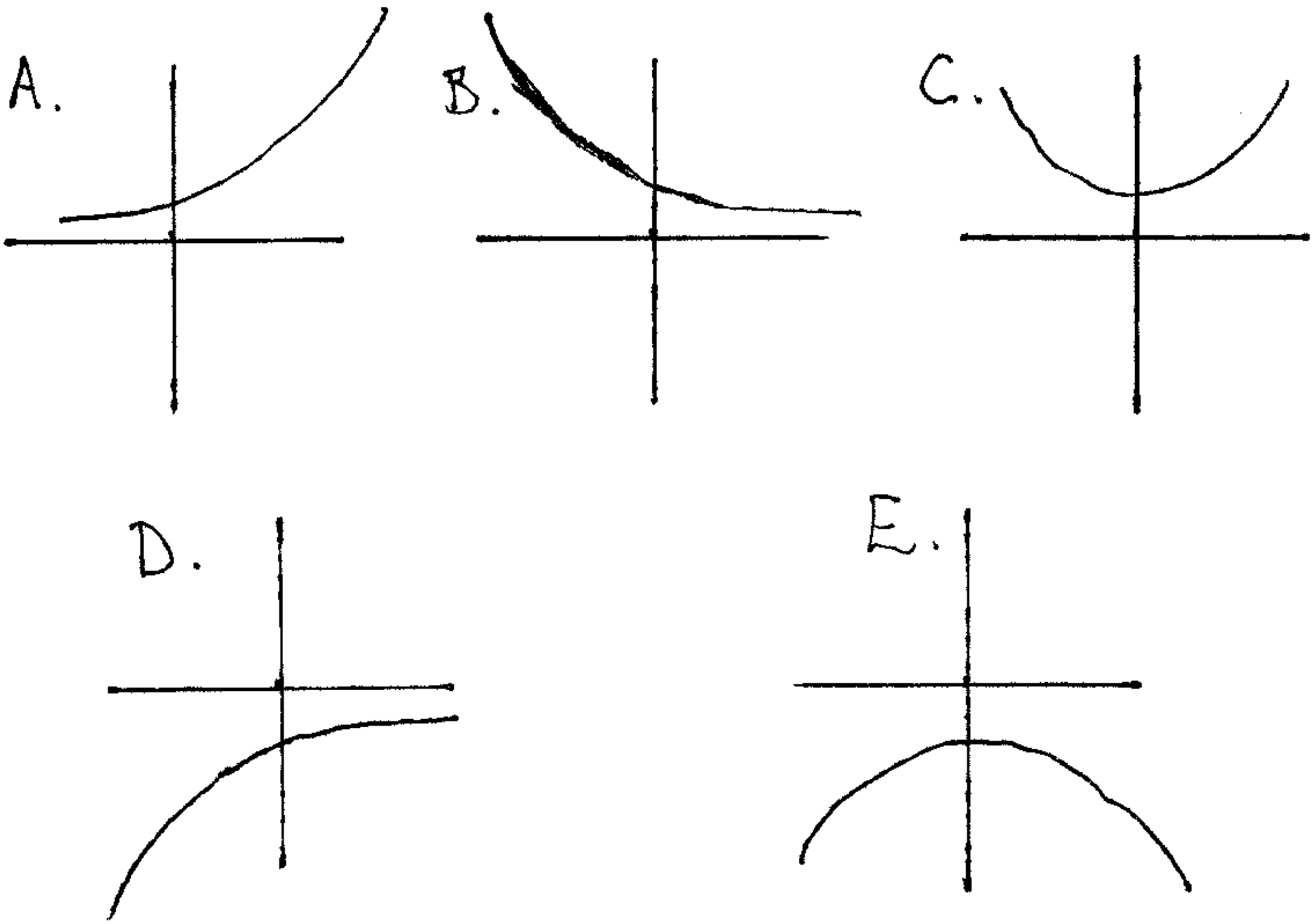
B. $\lim_{x\to 2} f(x)g(x) = \infty$

C. $\lim_{x\to 2} f(x)g(x) = 1$

D. $\lim_{x\to 2} f(x)g(x)$ is either 0 or ∞

E. The limit may or may not exist. If it exists it can be any number.

10. The graph of the function $f(x) = (1/2)^x$ looks most like:



11. $\lim_{t \to 0} t \sin(1/t^2) =$

A. 1

B. $\frac{1}{2}$

C. 0

D. ∞

E. The limit does not exist.

12. Use linear approximation to estimate the value of $\cosh(1.1)$ rounded to 2 decimal places. Note that $\cosh(1) \approx 1.54$ and $\sinh(1) \approx 1.18$.

A. 1.42

B. 1.66

C. 1.30

D. 1.06

E. 1.58

13. Find the number c that satisfies the conclusion of the Mean Value Theorem on $[0, 9]$ if $f(x) = 2\sqrt{x}$.

A. $c = \frac{9}{4}$

B. $c = 0$

C. $c = \frac{1}{4}$

D. $c = 5$

E. No such number exists in $(0, 9)$

14. Compute the limit

$$\lim_{x \to 0} \frac{e^{2x} - 1}{\tan x}$$

A. -1

B. 0

C. 1

D. 2

E. -2

15. According to the Intermediate Value Theorem, the function $f(x) = e^x + 3x - 4$ has a zero in the interval

A. $(-1, 0)$

B. $(0, 1)$

C. $(2, 4)$

D. $(1, 2)$

E. Both A. and B.

16. For which real number a does

$$\lim_{r \to 5} \frac{ar^2 + 25}{r - 5}, \quad \text{exist?}$$

A. $a = -2$

B. $a = -1$

C. $a = 1$

D. $a = 2$

E. $a = -5$

17. If

$$f(x) = \frac{x^2 e^{2x}}{(x^2 - 1)},$$

then $f'(x) =$

A. $\dfrac{x^3 e^{2x}\,(x^2 - x + 1)}{(x^2 - 1)^2}$

B. $\dfrac{2x e^{2x}\,(x^3 - x - 1)}{(x^2 - 1)^2}$

C. $\dfrac{2x^3 e^{2x}\,(x^3 - x - 1)}{(x^2 - 1)^2}$

D. $\dfrac{2x^2 e^{2x}\,(x^3 - x - 1)}{(x^2 - 1)^2}$

E. $\dfrac{2x e^{2x}\,(x^3 - x - 1)}{(x^2 - 1)}$

18. If $f(x) = x^{\sqrt{x}}$, then $f'(4) =$

A. $4 \ln 4 + 4$

B. $16 \ln 4 + 16$

C. $4 \ln 4 + 8$

D. $16 \ln 4 + 8$

E. $4 \ln 4 + 16$

19. Sand is dumped at a rate of 24 ft^3/min onto a pile whose shape is a cone whose base diameter and height are always equal. How fast is the height of the pile increasing (in ft/min) when the pile is 8 ft high? ($V = \frac{1}{3}\pi r^2 h$)

A. $\frac{8}{3\pi}$

B. $\frac{3}{2\pi}$

C. $\frac{2}{\pi}$

D. $\frac{3}{4\pi}$

E. $\frac{4}{3\pi}$

20. A cylindrical soup can is to have a volume of 1 cubic foot. The material for the side of the can costs $\$5/ft^2$, while the material for the top and bottom costs $\$2/ft^2$. What is the radius of a can that minimizes the cost of manufacturing? (The volume of the can is $\pi r^2 h$, the area of the top is πr^2, and the area of the side is $2\pi r h$.)

A. $\left(\frac{5}{4\pi}\right)^{1/3}$

B. $\left(\frac{5}{8\pi}\right)^{1/3}$

C. $2\sqrt{5\pi} + 20$

D. $10\pi + 20$

E. $1/5$

21. If

$$f(x) = \int_5^{\sin^2(x)} g(t)dt$$

then $f'(x) =$

A. $g(\sin^2(x))g'(x)$

B. $g(\sin^2(x))$

C. $g(\sin^2(x))2\sin(x)\cos(x)$

D. $g'(\sin^2(x))$

E. $g'(\sin^2(x))2\sin(x)\cos(x)$

22. Calculate

$$\int_0^{\pi/4} \frac{1}{\cos^2(x)\sqrt{1+2\tan(x)}}dx$$

A. 0

B. $\sqrt{3}-1$

C. 1

D. $\sqrt{3}$

E. 2