MA 16100
FINAL EXAM Green
December 12, 2018

NAME _____ YOUR TA'S NAME _____

STUDENT ID # _____ RECITATION TIME _____

Write the following in the TEST/QUIZ NUMBER boxes: **00** (and blacken in the appropriate digits below the boxes)

You must use a #2 pencil on the mark–sense sheet (answer sheet).On the mark–sense sheet, fill in your TA's name and the COURSE number. Fill in your NAME and STUDENT IDENTIFICATION NUMBER and blacken in the appropriate spaces. Fill in your four-digit SECTION NUMBER. If you do not know your section number, ask your TA. Sign the mark–sense sheet.

There are **25** questions, each worth 8 points. Blacken in your choice of the correct answer in the spaces provided for questions 1–25. Do all your work in this exam booklet. Use the back of the test pages for scrap paper. Turn in both the mark–sense sheet and the exam booklet when you are finished.

If you finish the exam before 9:50, you may leave the room after turning in the scantron sheet and the exam booklet. You may not leave the room before 8:20. If you don't finish before 9:50, you MUST REMAIN SEATED until your TA comes and collects your scantron sheet and your exam booklet.

EXAM POLICIES

1. Students may not open the exam until instructed to do so.
2. Students must obey the orders and requests by all proctors, TAs, and lecturers.
3. No student may leave in the first 20 min or in the last 10 min of the exam.
4. Books, notes, calculators, or any electronic devices are not allowed on the exam, and they should not even be in sight in the exam room. Students may not look at anybody else's test, and may not communicate with anybody else except, if they have a question, with their TA or lecturer.
5. After time is called, the students have to put down all writing instruments and remain in their seats, while the TAs will collect the scantrons and the exams.
6. Any violation of these rules and any act of academic dishonesty may result in severe penalties. Additionally, all violators will be reported to the Office of the Dean of Students.

I have read and understand the exam rules stated above:

STUDENT NAME: _____

STUDENT SIGNATURE: _____

1. Suppose the domain of $f(x)$ is $[0, \infty)$. If

$$g(x) = 1 - f(x+1)$$

then what is the domain of the function g ?

A. $(-\infty, 1]$
B. $(-\infty, -1]$
C. $[-1, \infty)$
D. $[0, \infty)$
E. $[1, \infty)$

2. If $f(x) = \dfrac{x+5}{x+1}$, simplify the expression

$$\frac{f(x) - f(3)}{x - 3}$$

A. $\dfrac{-2}{x-3}$
B. $\dfrac{-1}{x+1}$
C. $\dfrac{-4}{(x+1)^2}$
D. $\dfrac{3-x}{x+1}$
E. $\dfrac{x+3}{x-3}$

3. The curve

$$y = \frac{1 - 2x}{x^3 - 1}$$

has one horizontal asymptote, $y = h$, and one vertical asymptote, $x = k$. What is $h + k$?

A. 1

B. 3

C. -1

D. $\frac{1}{2}$

E. $\frac{3}{2}$

4. Consider the function

$$f(x) = \begin{cases} -1 & \text{if } x < 0 \\ 0 & \text{if } x = 0 \\ 1 & \text{if } x > 0 \end{cases}$$

Which of the following limits exist?

(i) $\lim_{x \to 0^+} f(x)$ (ii) $\lim_{x \to 0} f(x)$ (iii) $\lim_{x \to 0} |f(x)|$

A. (i) only

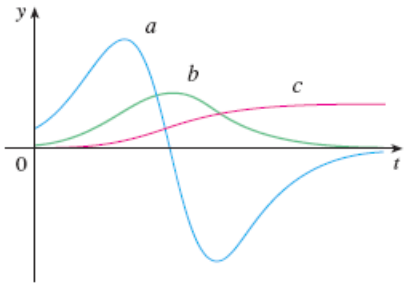
B. (i) and (iii) only

C. (ii) and (iii) only

D. (ii) only

E. (iii) only

5. The figure shows the graphs of three functions. One is the position function of a car, one is the velocity of the car, and one is its acceleration. Find the correct choice.



A. $a =$ acceleration, $b =$ position, $c =$ velocity

B. $a =$ velocity, $b =$ position, $c =$ acceleration

C. $a =$ acceleration, $b =$ velocity, $c =$ position

D. $a =$ position, $b =$ velocity, $c =$ acceleration

E. $a =$ velocity, $b =$ acceleration, $c =$ position

6. Find the limit.

$$\lim_{x \to 2} \sqrt{\frac{x^2 - 4}{x - 2}}$$

A. Does not exist

B. 0

C. 4

D. 2

E. ∞

7. Find the limit.

$$\lim_{x \to 0} \frac{\csc(3x)}{\cot x}$$

A. $\frac{1}{3}$

B. ∞

C. 1

D. Does not exist

E. 3

8. Find the limit.

$$\lim_{x \to \infty} \left(1 + \frac{3}{x}\right)^{4x}$$

A. e^4

B. 4

C. e^{12}

D. 12

E. $e^{3/4}$

9. The half life of a certain substance is 1000 years. How much of a sample weighing 100 kg remains after 100 years?

A. 10 kg

B. $10\ln(2)$ kg

C. 95 kg

D. $\dfrac{100}{\sqrt[10]{2}}$ kg

E. $10\sqrt[10]{2}$ kg

10. If $f(x) = \dfrac{x^2+3x-4}{x^2-1}$ then $f'(x) =$

A. $\dfrac{-3x^2-10x-3}{x^4-2x^2+1}$

B. $\dfrac{-3}{x^2-2x+1}$

C. $\dfrac{2x+3}{2x}$

D. $\dfrac{9x^2-10x-3}{x^4-2x^2+1}$

E. $\dfrac{-3}{x^2+2x+1}$

11. If $f(x) = \log_{10}(x)$ then $f'(e) =$

A. 1

B. $\dfrac{1}{10}$

C. $\dfrac{1}{e}$

D. $\dfrac{1}{e\ln(10)}$

E. $\dfrac{e}{10}$

12. If $f(x) = (1-x^2)\sin^{-1}(x)$ then $f'(x) =$

A. $\dfrac{-2x}{\sqrt{1-x^2}}$

B. $\sqrt{1-x^2} - 2x\sin^{-1}(x)$

C. $1 - 2x\sin^{-1}(x)$

D. $-2x\cos^{-1}(x)$

E. $\dfrac{1-x^2}{1+x^2} - 2x\sin^{-1}(x)$

13. If for differentiable functions $f(x)$ and $g(x)$ we have $f(2) = 2, f(3) = 5, f'(2) = -3, f'(3) = 7$ and $g(2) = 3, g(3) = 2, g'(2) = -1, g'(3) = -3$, then

$$\frac{d}{dx} f(g(x))\Big|_{x=2} =$$

A. 3

B. -7

C. 7

D. 21

E. -9

14. If $\cos(x + y) = y^2 \sin(x)$ then $\dfrac{dy}{dx} =$

A. $\dfrac{-y^2 \cos(x)}{2y \sin(x) + \sin(x + y)}$

B. $\dfrac{-y^2 \cos(x) - \sin(x + y)}{2y \sin(x) + \sin(x + y)}$

C. $\dfrac{-y^2 \cos(x) - \sin(x + y)}{2y \sin(x)}$

D. $\dfrac{2y \sin(x) + y^2 \cos(x)}{-\sin(x + y)}$

E. Cannot be determined, since one cannot solve for y.

15. Water is poured into a conical paper cup at the rate of 4 cubic centimeters per second. If the cup is 18 cm tall and the top has a radius of 6 cm, how fast is the water level rising when the water is 9 cm deep? (Volume of the cone: $V = \frac{1}{3}\pi r^2 h$).

A. $\dfrac{4}{9\pi}$ cm/s

B. $\dfrac{\pi}{3}$ cm/s

C. $\dfrac{4\pi}{9}$ cm/s

D. $\dfrac{4}{81\pi}$ cm/s

E. $\dfrac{9\pi}{4}$ cm/s

16. By linearization (differentials), the approximate value of $\sqrt[4]{17}$ is

A. 2

B. $\frac{63}{32}$

C. $\frac{31}{16}$

D. $\frac{33}{16}$

E. $\frac{65}{32}$

17. The intervals on which the function $f(x) = x^4 - 4x^3 + 4x^2$ is decreasing and those on which it is increasing are given by

A. Increases on $(-\infty, 0) \cup (1, 2)$ and decreases on $(0, 1) \cup (2, \infty)$

B. Decreases on $(-\infty, 0) \cup (1, 3)$ and increases on $(0, 1) \cup (3, \infty)$

C. Decreases on $(-\infty, 0) \cup (1, 2)$ and increases on $(0, 1) \cup (2, \infty)$

D. Decreases on $(-\infty, 2)$ and increases on $(2, \infty)$

E. None of the above

18. The minimum value of the function $f(x) = \frac{3}{8}x^4 + x^3$ is

A. -4

B. $-\frac{3}{2}$

C. -2

D. 0

E. $-\frac{5}{8}$

19. A box with square base and open top must have a volume of 40 cubic inches. What are the dimensions of the box that minimize the amount of material to build it?

A. $\sqrt[3]{10} \times \sqrt[3]{10}$ base with $4\sqrt[3]{10}$ height.

B. 2×2 base with 10 height.

C. $\sqrt{10} \times \sqrt{10}$ base with 4 height.

D. $2\sqrt[3]{10} \times 2\sqrt[3]{10}$ base with $\sqrt[3]{10}$ height.

E. $\sqrt{5} \times \sqrt{5}$ base with 8 height.

20. Find an antiderivative of the function $f(x) = \dfrac{(x-1)^2}{x}$

A. $\dfrac{2(x-1)^3}{3x^2}$

B. $\dfrac{x^2-1}{x^2}$

C. $\dfrac{1}{2}x^2 - 2x + \ln|x|$

D. $\dfrac{x(x-1)^3}{3}$

E. $\dfrac{(x-1)^2(2x+1)}{6}$

21. Estimate the area under the graph of $f(x) = \sin(x)$ from $x = 0$ to $x = \dfrac{\pi}{2}$ using three approximating rectangles and left endpoints. In other words, find the left Riemann sum, L_3.

A. $\dfrac{\pi(3+\sqrt{3})}{12}$

B. $\dfrac{\pi(3+\sqrt{3})}{2}$

C. $\dfrac{\pi(1+\sqrt{3})}{12}$

D. $\dfrac{\pi(1+\sqrt{3})}{4}$

E. $\dfrac{\pi(3+\sqrt{3})}{4}$

22. If a particle is moving at the speed $v(t) = \dfrac{1}{t^2+1}$, then the distance it covers from time $t = 1$ to $t = \sqrt{3}$ is

A. $\dfrac{\pi}{3}$

B. $\dfrac{\pi}{4}$

C. $\dfrac{\pi}{2}$

D. π

E. $\dfrac{\pi}{12}$

23. Suppose

$$g(x) = \int_4^{x^2} e^{t^2}\, dt$$

Find $g'(2)$, the derivative of $g(x)$ at $x = 2$.

A. $4e^{16}$

B. 0

C. $4e^4$

D. e^4

E. e^{16}

24. Evaluate the indefinite integral

$$\int \left(\sinh x + \sinh^3 x\right)\, dx$$

Hint: $\cosh^2 x - \sinh^2 x = 1$

A. $\cosh x + \frac{1}{4}\sinh^3 x \tanh x + C$

B. $\frac{1}{2}(\cosh^2 x - 1) + \frac{1}{4}\sinh^4 x + C$

C. $\frac{1}{3}\cosh^3 x + C$

D. $\cosh x(1 + \sinh^2 x) + C$

E. $\cosh x + \frac{1}{4}(\cosh^2 x - 1)^2 + C$

25. $\displaystyle\int_0^{\pi} \sin t\ \sqrt{1+\cos t}\ dt =$

A. $-\dfrac{2\pi\sqrt{\pi}}{3}$

B. $\dfrac{4\sqrt{2}}{3}$

C. $\dfrac{4}{3}$

D. 0

E. $-\sqrt{\pi}$