

MA 16100  
FINAL EXAM (Version 11)  
(December 16, 2021)

NAME \_\_\_\_\_ YOUR TA's NAME \_\_\_\_\_

PUID # \_\_\_\_\_ RECITATION TIME \_\_\_\_\_

Write the following in the TEST/QUIZ NUMBER boxes: 11 (and blacken the appropriate digits below the boxes). You must use a #2 pencil on the mark-sense sheet (answer sheet). On the mark-sense sheet, fill in your TA's NAME and the COURSE number. Fill in YOUR NAME and TEN-DIGIT PUID NUMBER and blacken the appropriate spaces. Fill in your FOUR-DIGIT SECTION NUMBER. If you do not know your section number, ask your TA. Sign the mark-sense sheet.

There are 25 questions, each worth 8 points, for a total of 200 points. Blacken in your choice of the correct answer in the spaces provided for questions 1-25. Do all your work in the exam booklet, and also circle the answers in the exam booklet in case of a lost mark-sense sheet. Use the back of the test pages for scrap paper. Turn in both the mark-sense sheet and the exam booklet when you are finished.

If you finish the exam before 2:50pm you may leave the room after turning in the mark-sense sheet and exam booklet. You may not leave the room before 1:20pm. If you don't finish before 2:50pm you MUST REMAIN SEATED until your TA comes and collects your mark-sense sheet and your exam booklet.

EXAM RULES

1. Students may not open the exam until instructed to do so
2. Students must obey the orders and requests by all proctors, TA, and lecturers.
3. No student may leave in the first 20 min or in the last 10 min of the exam.
4. Books, notes, calculators, phones, or any electronic devices are not allowed on the exam, and they should not even be in sight in the exam room. Students may not look at anybody else's test, and may not communicate with anybody else except, if they have a question, with their TA or lecturer.
5. After time is called, the students have to put down all writing instruments and remain in their seats, while the TAs collect the mark-sense sheets and exams.
6. Any violation of these rules or any act of academic dishonesty may result in severe penalties. Additionally, all violators will be reported to the Office of the Dean of Students.

I have read and understand the exam rules stated above:

STUDENT NAME \_\_\_\_\_

STUDENT SIGNATURE \_\_\_\_\_

1. Find  $\int_0^1 \sqrt{3x+1} dx$

- A.  $\frac{13}{9}$
- B.  $\frac{14}{9}$
- C.  $\frac{11}{9}$
- D.  $\frac{8}{9}$
- E.  $\frac{10}{9}$

2. For the function  $f(x) = x$  on the interval  $[2, 8]$ , find the left Riemann sum with  $n = 3$  subintervals of equal length.

- A. 16
- B. 22
- C. 24
- D. 18
- E. 12

3. Find the integral:

$$\int_0^1 \frac{x^2}{(x^3 + 4)^2} dx$$

- A.  $\frac{1}{60}$
- B.  $\frac{1}{12}$
- C.  $\frac{1}{30}$
- D.  $\frac{1}{15}$
- E.  $\frac{1}{20}$

4. Find the integral:

$$\int_1^4 \frac{(\sqrt{x} - 1)^7}{\sqrt{x}} dx$$

- A.  $\frac{1}{16}$
- B.  $\frac{1}{2}$
- C.  $\frac{1}{8}$
- D.  $\frac{1}{32}$
- E.  $\frac{1}{4}$

5. Find the integral:

$$\int_0^{\pi/3} \cos^2 x \sin x \, dx$$

- A.  $\frac{17}{24}$
- B.  $\frac{5}{24}$
- C.  $\frac{11}{24}$
- D.  $\frac{7}{24}$
- E.  $\frac{13}{24}$

6. Find

$$\frac{d}{dx} \int_1^{\ln(2x)} \sqrt{e^t - 1} \, dt$$

when  $x = 13$ .

- A.  $\frac{7}{13}$
- B.  $\frac{5}{13}$
- C.  $\frac{4}{13}$
- D.  $\frac{6}{13}$
- E.  $\frac{8}{13}$

7. A certain population of bacteria is growing exponentially. At time  $t = 0$  there are 100 bacteria and at time  $t = 1$  there are 110 bacteria. At what time will there be 120 bacteria present?

A.  $\ln\left(\frac{1.2}{1.1}\right)$

B.  $\frac{\ln 1.1}{1.2}$

C.  $\ln\left(\frac{1.1}{1.2}\right)$

D.  $\frac{\ln 1.1}{\ln 1.2}$

E.  $\frac{\ln 1.2}{\ln 1.1}$

8. Simplify:

$$\ln\left(\frac{(x^7 x^5)^{3/2}}{(x^{15})^{2/3}}\right)$$

A.  $5 \ln x$

B.  $9 \ln x$

C.  $8 \ln x$

D.  $6 \ln x$

E.  $7 \ln x$

9. Find  $f'(0)$  if  $f(x) = (2x + 1)^3(x + 2)^2$

- A. 38
- B. 23
- C. 28
- D. 33
- E. 18

10. Find  $f'(0)$  if

$$f(x) = \frac{(2x - 1)^3}{(2x + 1)^2}$$

- A. 10
- B. 4
- C. 2
- D. 8
- E. 6

11. Find  $f'(0)$  if  $f(x) = e^{2\sin 3x}$

- A. 3
- B. 0
- C. 2
- D. 9
- E. 6

12. Find  $f'(0)$  if

$$f(x) = \ln \left( \frac{(x+1)^2}{(x+2)(x+4)} \right)$$

- A.  $\frac{9}{4}$
- B.  $\frac{5}{4}$
- C.  $\frac{1}{4}$
- D.  $\frac{7}{4}$
- E.  $\frac{3}{4}$

13. Find  $f'(1)$  if  $f(x) = \tan^{-1}(3x^2)$

- A.  $\frac{4}{5}$
- B.  $\frac{1}{10}$
- C.  $\frac{3}{10}$
- D.  $\frac{3}{5}$
- E.  $\frac{1}{5}$

14. Find the second derivative  $f''(1)$  if  $f(x) = \frac{x^2}{x+1}$

- A.  $\frac{1}{8}$
- B.  $\frac{3}{4}$
- C.  $\frac{3}{8}$
- D.  $\frac{1}{4}$
- E.  $\frac{5}{8}$



15. Use implicit differentiation to find  $\frac{dy}{dx}$  at the point (1, 2):

$$x^2 + xy + y^2 = 7$$

- A.  $-\frac{4}{5}$
- B.  $-\frac{3}{5}$
- C.  $-\frac{7}{10}$
- D.  $-\frac{3}{10}$
- E.  $-\frac{9}{10}$

16. Let  $f(x) = \frac{1}{x}$ . Use the definition of the derivative to find  $f'(3)$ . When you simplify the terms inside the limit, you get:

- A.  $\lim_{h \rightarrow 0} \left( -\frac{1}{9 + 5h} \right)$
- B.  $\lim_{h \rightarrow 0} \left( -\frac{1}{9 + 3h} \right)$
- C.  $\lim_{h \rightarrow 0} \left( -\frac{1}{9 + 2h} \right)$
- D.  $\lim_{h \rightarrow 0} \left( -\frac{1}{9 + h} \right)$
- E.  $\lim_{h \rightarrow 0} \left( -\frac{1}{9 + 4h} \right)$

17. Sand is poured out of a bag onto the ground and accumulates in a conical pile with a radius that is always two times its height. Suppose that at the moment when the pile is 20 cm high the height of the pile is increasing at 3 cm/sec. At what rate is the volume of the conical pile of sand increasing at that moment? The volume of a cone is  $V = \frac{1}{3}\pi r^2 h$ .



- A.  $5600\pi$  cm<sup>3</sup>/sec  
B.  $4400\pi$  cm<sup>3</sup>/sec  
C.  $4000\pi$  cm<sup>3</sup>/sec  
D.  $5200\pi$  cm<sup>3</sup>/sec  
E.  $4800\pi$  cm<sup>3</sup>/sec
18. If the **first derivative** of  $f(x)$  is
- $$f'(x) = (x + 1)^3(x - 4)^2(x + 3)^4(x - 1)^5,$$
- which of the points  $-1, -3, 1$  are local maxima?
- A.  $-1$  and  $-3$   
B.  $1$  only  
C.  $-1$  only  
D.  $-3$  and  $1$ .  
E.  $-3$  only

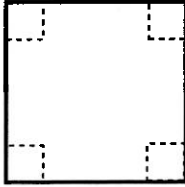
19. For the graph of  $y = x^3 - 3x^2$  on the interval  $[1.5, 1.7]$ , which of these is a correct description?

- A. The graph is decreasing and concave down on this interval.
- B. The graph is increasing and concave up on this interval.
- C. The graph is increasing and concave down on this interval.
- D. The graph is decreasing and concave up on this interval.
- E. None of the above.

20. The rational function  $f(x) = \frac{(x^2 - 4)^3}{x^3 + 2x^2}$  has

- A. 2 *vertical* asymptotes and 1 *horizontal* asymptote
- B. 1 *vertical* asymptote and 1 *horizontal* asymptote
- C. 2 *vertical* asymptotes and 2 *horizontal* asymptotes
- D. 2 *vertical* asymptotes and no *horizontal* asymptote
- E. 1 *vertical* asymptote and no *horizontal* asymptote

21. A square piece of cardboard has sides of length 2 ft. Squares with sides of length  $x$  are cut out of each corner, and the resulting piece of cardboard is then folded into a box without a lid. Find the volume of the largest box that can be formed in this way.



- A.  $\frac{16}{27}$  ft<sup>3</sup>  
B.  $\frac{19}{27}$  ft<sup>3</sup>  
C.  $\frac{17}{27}$  ft<sup>3</sup>  
D.  $\frac{20}{27}$  ft<sup>3</sup>  
E.  $\frac{14}{27}$  ft<sup>3</sup>
22. Use linear approximation and the fact that  $4^{3/2} = 8$  to estimate  $(3.98)^{3/2}$ .
- A. 7.92  
B. 7.94  
C. 7.93  
D. 7.95  
E. 7.96

23. Find the limit:

$$\lim_{x \rightarrow 3} \frac{x^2 - 9}{\sqrt{2x + 3} - 3}$$

- A. 0
- B. 2
- C. 18
- D. 9
- E. 4

24. Find the limit:

$$\lim_{x \rightarrow 0} \frac{e^{3x} - 3x - 1}{x^2}$$

- A.  $\frac{11}{2}$
- B.  $\frac{9}{2}$
- C.  $\frac{13}{2}$
- D.  $\frac{7}{2}$
- E.  $\frac{15}{2}$

25. If  $F(x)$  is the antiderivative of  $f(x) = 2x + \frac{2}{x^2}$  where  $F(1) = 2$ , then  $F(2) =$

- A. 6
- B. 7
- C. 5
- D. 3
- E. 4