MA 161 & 161E **FINAL EXAM** **SPRING 2004**

Name ______________________ Student ID number ______________________

Recitation Instructor ______________________ Div-Sec ______________________

Recitation Time ______________________

Instructions:

1. Fill in all the information requested above and on the scantron sheet.
2. The exam has 25 problems, each worth 8 points, for a total of 200 points.
3. For each problem mark your answer on the scantron sheet and also circle it in this booklet. Use a number 2 pencil on the answer sheet. Be sure to fill in the circles for each of the answers of the 25 exam questions.
4. Work only on the pages of this booklet.
5. Books, notes, calculators are not to be used on this test.
6. At the end turn in your exam and scantron sheet to your recitation instructor.

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1. The tangent line to $f(x) = \dfrac{32}{x+2}$ at $x = 2$ is

A. $y = x + 6$

B. $y = 2x + 4$

C. $y = -8x + 8$

D. $y = -2x + 12$

E. $y = 8x - 8$

2. $\lim\limits_{x \to 0} \dfrac{\cos(2x) - 1}{x^2}$ equals

A. 0

B. 1

C. -1

D. 2

E. -2

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3. If $f(x) = \ln(x^3 + 2x^2)$ then $f'(2)$ equals

A. $5/8$

B. $4\ln 16$

C. $5/4$

D. $10/3$

E. $8\ln 16$

4. If $y\sin x + x\sin y = 20$, then $y'(x)$ equals

A. $\dfrac{-(y\sin x - \cos x)}{(\sin y - y\cos x)}$

B. $\dfrac{-(y\cos x + \sin y)}{(\sin x + x\cos y)}$

C. $\dfrac{-(x\sin y + \cos y)}{(\sin x + x\cos y)}$

D. $\dfrac{-(y\sin x + \cos x)}{(\sin y + y\cos x)}$

E. $\dfrac{-(x\sin y + \cos y)}{(\cos x + x\sin y)}$

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5. If $f(x) = \tan^{-1}(2x)$ then $f''(x)$ equals

A. $\dfrac{-4x}{(1+x^2)^2}$

B. $\dfrac{2}{1+4x}$

C. $\dfrac{-8}{(1+4x)^2}$

D. $\dfrac{2}{1+4x^2}$

E. $\dfrac{-16x}{(1+4x^2)^2}$

6. If $f(x) = \sin(\pi e^{x/2})$ then $f'(0)$ equals

A. 0

B. $\dfrac{-\pi}{2}$

C. $\dfrac{\pi}{2}$

D. $\dfrac{-\pi}{2}\, e^{1/2}$

E. $\dfrac{e^{1/2}}{2}$

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7. Find the absolute minimum value for

   $f(x) = x^3 - 3x$ on $[-3, 2]$.

   A. 2
   B. -2
   C. -1
   D. -20
   E. -18

8. The largest interval on which

   $f(x) = -2x^3 + 3x^2 + 12x$

   is increasing is

   A. $(-1, 4)$
   B. $(2, 4)$
   C. $(\frac{1}{2}, 2)$
   D. $(-1, \frac{1}{2})$
   E. $(-1, 2)$

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9. $\lim_{x \to 2} \dfrac{e^{x^2} - e^4}{x - 2}$ equals

A. e^4

B. $2e^4$

C. $e^4/4$

D. $8e^4$

E. $4e^4$

10. $\int_0^2 |x - 1|\, dx$ equals

A. $1/2$

B. 2

C. $3/2$

D. 1

E. $5/2$

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11. $\int_0^3 \frac{x}{\sqrt{x+1}}\,dx$ equals

A. $\frac{8}{3}$

B. $\frac{2}{3}$

C. 1

D. $\frac{4}{3}$

E. $\frac{5}{3}$

12. $\int_0^1 (1+x^2)^2 dx$ equals

A. $\frac{14}{25}$

B. $\frac{4}{3}$

C. 4

D. $\frac{28}{15}$

E. $\frac{7}{3}$

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13. Let $f(x) = x^3 - x^2 + 2x + 1$. If $x_0 = 1$ then the first approximation, x_1, to a root of $f(x)$ using Newton's method is

A. 0

B. 1/3

C. 2/3

D. 4/3

E. 2

14. Determine the number of vertical and horizontal asymptotes for

$$f(x) = \frac{x^4}{(x^2-1)(x^2+4)}.$$

A. 1

B. 2

C. 3

D. 4

E. 5

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15. If $f'(x) = 2x^3 + x$ and $f(0) = 4$ then $f(2)$ equals

A. 14

B. 12

C. 24

D. 18

E. 25

16. Determine b so that $f(x) = x^2 + \frac{b}{x^3}$ has an inflection point at $x = 1$.

A. 1

B. $-\frac{1}{6}$

C. $\frac{1}{3}$

D. $-\frac{1}{3}$

E. $\frac{1}{6}$

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17. Determine a so that

$$f(x) = \begin{cases} 5+x & \text{if } x \le 0, \\ \sqrt{x^2+a} & \text{if } x > 0 \end{cases}$$

is continuous at $x = 0$.

A. 0

B. 1

C. 25

D. 5

E. -5

18. Let $f(x)$ be a continuous function such that

$1 - x^2 \le f(x) \le 1 + x^2$.

Then $\int_0^1 f(x)dx$ must be in the interval

A. $\left[\frac{5}{6}, \frac{4}{3}\right]$

B. $\left[\frac{2}{3}, \frac{4}{3}\right]$

C. $\left[\frac{2}{3}, \frac{7}{6}\right]$

D. $\left[1, \frac{4}{3}\right]$

E. $\left[1, \frac{7}{6}\right]$

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19. Let

$$f(x) = \begin{cases} x^2 + 4x + 1 & \text{if } x < 0, \\ x^2 - 2x + 1 & \text{if } x \geq 0. \end{cases}$$

Which of the following statements is true?

A. f has a local minimum at $x = 0$

B. f has an inflection point at $x = 0$

C. f has a local maximum at $x = 0$

D. f is discontinuous at $x = 0$

E. f is differentiable at at $x = 0$

20. If $\int_0^x f(t)dt = x^4 - 3x^2 + x$ for all x then $f(1)$ equals

A. -1

B. 0

C. 1

D. 2

E. 3

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21. Let $f(x)$ be differentiable on $(-\infty, \infty)$ and let $F(x) = f(f(x))$. Then $F'(x)$ equals

A. $f'(f(x))$

B. $f'(f(x)) \cdot f'(x)$

C. $f'(f(x)) \cdot f(x)$

D. $f'(x) \cdot f(f'(x))$

E. $2f'(f(x))$

22. A kite is 100 ft above the ground and moves horizontally at a rate of 10 ft/sec. At what rate (in radians/sec) is the angle between the string and the ground changing when 200 ft of string has been let out?

A. $-1/10$

B. $-1/20$

C. $-1/30$

D. $-1/40$

E. $-1/50$

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23. A box with an open top is constructed from a square piece of cardboard 5 ft. wide by cutting out a square from each of the four corners and then bending up the sides. The box is then fitted with a lid cut from a second piece of cardboard. Assume the length of the sides of the squares cut out is x. Find the total surface area of the box, including the lid, as a function of x. $A(x) =$

A. $50 - 20x$

B. $50 - 10x - 3x^2$

C. $150 - 120x + 24x^2$

D. $150 - 60x + 6x^2$

E. $50 - 10x$

24. Let $g(x)$ be a differentiable function on $(-\infty, \infty)$ such that $g(5) = -5$ and $g(6) = 3$. Which of the following statements must be true?

I. There is c in the interval (5,6) with $g(c) = 8$.

II. There is c in the interval (5,6) with $g'(c) = 8$.

III. There is c in the interval (5,6) with $g(c) = 0$.

A. Just I

B. Just I and II

C. Just I and III

D. Just II and III

E. All three

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25. The largest area of an isosceles triangle that can be inscribed in a circle of radius 1 is

A. 1

B. $\frac{\sqrt{3}}{2}\,\pi$

C. $\frac{3\sqrt{3}}{4}$

D. $\sqrt{3}\,\pi$

E. $\frac{\sqrt{3}}{2}$

Notes: You can assume that $\sqrt{3} = 1.73$ and $\pi = 3.14$. An isosceles triangle has two sides of equal length.