MA 16100 Final Exam, Spring 2014, May 9

Name _____

10 -digit PUID number _____

Recitation Instructor _____

Recitation Section Number and Time _____

Instructions: MARK TEST NUMBER 01 ON YOUR SCANTRON

1. Do not open this booklet until you are instructed to.
2. Fill in all the information requested above and on the scantron sheet. On the scantron sheet fill in the little circles for your name, section number and PUID.
3. This booklet contains 24 problems, equally weighted.
4. For each problem mark your answer on the scantron sheet and also **circle it in this booklet**.
5. Work only on the pages of this booklet.
6. Books, notes, calculators or any electronic device are not allowed during this test and they should not even be in sight in the exam room. You may not look at anybody else's test, and you may not communicate with anybody else, except, if you have a question, with your instructor.
7. You are not allowed to leave during the first 20 and the last 10 minutes of the exam.
8. When time is called at the end of the exam, put down your writing instruments and remain seated. The TAs will collect the scantrons and the booklets.

1. A rectangular cardboard box with no top has a rectangular base so that one side is twice as long as the other. If the box must have a volume of $\frac{4}{3}$ m^3, what should the height of the box be to minimize the amount of cardboard used?

   A. $\frac{1}{\sqrt{2}}$ m

   B. $\frac{2}{\sqrt[3]{3}}$ m

   C. $\left(\frac{2}{3}\right)^{\frac{1}{3}}$ m

   D. $\frac{2}{3}$ m

   E. $\frac{2\sqrt{2}}{3}$ m

2. Use a linear approximation (or differentials) to estimate $(1.99)^4$.

   A. 15.64

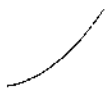
   B. 15.66

   C. 15.62

   D. 15.70

   E. 15.68

3. Which of the following pictures most resembles the graph $y = x^3 - 12x + 2$ between $x = 1$ and $x = 3$?



(A)

(B)

(C)

(D)

(E)

4. Find $M = \lim_{x \to \pi^-} \dfrac{1 + 2\sin 2x}{\sin x}$ and $P = \lim_{x \to \pi^+} \dfrac{1 + 2\sin 2x}{\sin x}$.

A. $M = P = -\infty$
B. $M = -\infty,\ P = \infty$
C. $M = \infty,\ P = -\infty$
D. $M = P = 4$
E. $M = -4, P = 4$

5. Which are true?

(I) If f is continuous on $[a, b]$ and differentiable on (a, b), then there is a c in (a, b) such that $f'(c) = \dfrac{f(b) - f(a)}{b - a}$.

(II) If f is continuous on $[a, b]$, differentiable on (a, b) and $f(a) < p < f(b)$, then there is a d in (a, b) such that $f'(d) = p$.

A. Only (II) is necessarily true
B. Both (I) and (II) are necessarily true
C. Neither is necessarily true
D. Both (I) and (II) are true, but only if f is everywhere positive
E. Only (I) is necessarily true

6. Find the constants a and b such that

$$F(x) = \begin{cases} \dfrac{x^2 - x}{x - 1}, & x < 1 \\ ax^2 + bx + 1, & 1 \leq x < 2 \\ x + a, & x \geq 2 \end{cases}$$

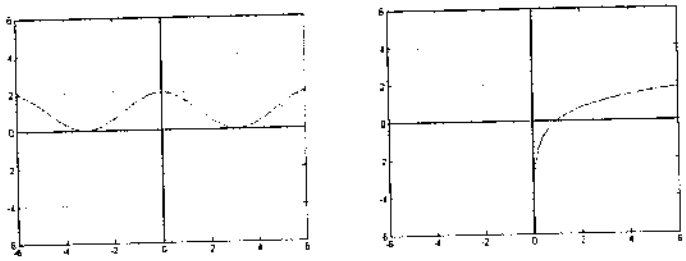
is continuous for all x.

A. $a = 3, b = -1$
B. $a = -1, b = 3$
C. $a = 2, b = 0$
D. $a = 1, b = -1$
E. $a = 1, b = 1$

7. If $\log_3 x = 1/2$ then $x =$

A. $\dfrac{1}{8}$
B. $e^{2/3}$
C. $\sqrt{3}$
D. $e^{3/2}$
E. $\dfrac{3}{2}$

8. Which functions are graphed below?



A. $3 - \sin x$ and e^x

B. $1 + \cos x$ and e^x

C. $1 + \sin x$ and e^x

D. $1 + \cos x$ and $\ln x$

E. $1 + \sin x$ and $\ln x$

9. The distance, s, between two points (x_1, y_1) and (x_2, y_2) is $s = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$. Find the point on $y = \sqrt{x^2 + 4}$ that is closest to (2,0).

A. $(2, \sqrt{8})$

B. $(\sqrt{2}, \sqrt{6})$

C. $(0, 2)$

D. $(1/2, \sqrt{17}/2)$

E. $(1, \sqrt{5})$

10. Compute $\int_0^1 (3x+1)(x-1)dx$.

A. -1
B. -2
C. 1
D. 2
E. 0

11. Find the absolute maximum value and the absolute minimum value of $f(x) = x^2 - 2x + 3$ on the interval $[0, 3]$.

A. The absolute maximum value is 3 and the absolute minimum value is 0
B. The absolute maximum value is 3 and the absolute minimum value is 2
C. The absolute maximum value is 6 and the absolute minimum value is 0
D. The absolute maximum value is 6 and the absolute minimum value is 2
E. The absolute maximum value is 6 and the absolute minimum value is 3

12. A sample of a radioactive element initially has mass of 24 gm. After 2 minutes the sample of that element has mass of 2 gm. When (in minutes) is the mass equal to 4 gm?

A. $\frac{2\ln 6}{\ln 12}$
B. $\frac{2\ln 12}{\ln 6}$
C. $\frac{\ln 6}{\ln 12}$
D. $\frac{3\ln 8}{\ln 3}$
E. $\frac{3\ln 6}{2\ln 2}$

13. What is the domain of $f(x) = \sqrt{4x - x^2} + \ln(1 - x)$?

A. $[0, 1) \cup (1, 4]$
B. $(1, 4]$
C. $(0, 1)$
D. $[0, 1)$
E. $[1, 4]$

14. Evaluate $\lim_{x\to 0^+}\left(\frac{1}{x}-\frac{1}{e^x-1}\right)$.

A. 2
B. 0
C. ∞
D. $-\infty$
E. $\frac{1}{2}$

15. Compute $\int_0^{\frac{\pi}{2}} \frac{d}{dx}(x^4\sin(x))dx$.

A. $\frac{\pi}{2}$
B. 0
C. 1
D. $\frac{\pi^3}{8}$
E. $\frac{\pi^4}{16}$

16. Find the derivative of

$$g(x) = \int_{\cos x}^{1} t^2 dt$$

A. $-2\cos x$
B. $\cos^2 x \sin x$
C. $-\cos^2 x \sin x$
D. $-2\cos x \sin x$
E. $2\cos x \sin x$

17. The slope of the line tangent to the graph of

$$y^2 - 2x^3 + xy^4 + 13 = 0$$

at $(2, 1)$ is

A. $\frac{12}{5}$
B. $\frac{23}{10}$
C. $\frac{18}{5}$
D. $-\frac{5}{18}$
E. -13

18. Let $h(x) = f(g(x))$, where $g(1) = 2$, $g'(1) = 3$, $f(1) = 4$, $f'(1) = 5$, $f(2) = 6$, and $f'(2) = 7$. Then $h'(1) =$

A. 7
B. 12
C. 15
D. 21
E. 5

19. Compute $\int_{\ln \frac{\pi}{4}}^{\ln \frac{\pi}{2}} e^x \cos e^x dx$.

A. 1
B. $\frac{\sqrt{3}+1}{2}$
C. $1-\sqrt{2}$
D. $\frac{\sqrt{3}-1}{2}$
E. $\frac{2-\sqrt{2}}{2}$

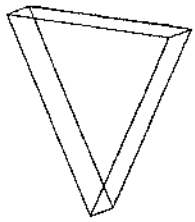
20. Find the derivative of $y = (\ln(x^2 + x + 2))^2$ when $x = 1$.

A. $\frac{1}{2}\ln 4$
B. $\frac{3}{2}\ln 4$
C. $2\ln 4$
D. $\frac{5}{2}\ln 4$
E. $\ln 4$

21. Find the derivative of $y = \frac{x}{x^2 + 3}$ when $x = 1$.

A. $\frac{7}{8}$
B. $\frac{5}{8}$
C. $\frac{3}{8}$
D. $\frac{9}{8}$
E. $\frac{1}{8}$

22. A trough is 4 ft long and its ends have the shape of isosceles triangles that are 20 ft across at the top and have a height of 20 ft. If the trough is being filled with water at a rate of 16 cubic feet per minute, how fast is the water level rising when the water is 2 feet deep?



A. 4 feet per minute
B. 3 feet per minute
C. 1 feet per minute
D. 5 feet per minute
E. 2 feet per minute

23. $\int x^3(1-2x^4)^{1/4}dx =$

A. $\frac{2}{5}(1-2x^4)^{5/4}+C$
B. $\frac{4}{5}(1-2x^4)^{5/4}+C$
C. $-\frac{1}{8}(1-2x^4)^{5/4}+C$
D. $-\frac{5}{32}(1-2x^4)^{5/4}+C$
E. $-\frac{1}{10}(1-2x^4)^{5/4}+C$

24. Compute $\int \frac{(\ln(x))^3}{3x} dx$.

A. $\frac{(\ln(x))^2}{6x} + C$

B. $\frac{(\ln(x))^4}{3x} + C$

C. $(\ln(x))^2 + C$

D. $\frac{(\ln(x))^4}{6} + C$

E. $\frac{(\ln(x))^4}{12} + C$