MA 16100
FINAL EXAM Form 01
May 1, 2019

NAME ______________________ YOUR TA'S NAME ______________________

STUDENT ID # ____________________ RECITATION TIME ____________________

Be sure the paper you are looking at right now is GREEN! Write the following in the TEST/QUIZ NUMBER boxes (and blacken in the appropriate spaces below the boxes): **01**

You must use a #2 pencil on the mark–sense sheet (answer sheet). On the mark–sense sheet, fill in your TA's name and the COURSE number. Fill in your NAME and STUDENT IDENTIFICATION NUMBER and blacken in the appropriate spaces. Fill in your four-digit SECTION NUMBER. If you do not know your section number, ask your TA. Sign the mark–sense sheet.

There are **25** questions, each worth 8 points. Blacken in your choice of the correct answer in the spaces provided for questions 1–25. Do all your work in this exam booklet. Use the back of the test pages for scrap paper. Turn in both the scantron and the exam booklet when you are finished.

If you finish the exam before 8:50, you may leave the room after turning in the scantron sheet and the exam booklet. You may not leave the room before 7:20. If you don't finish before 8:50, you MUST REMAIN SEATED until your TA comes and collects your scantron sheet and your exam booklet.

EXAM POLICIES

1. Students may not open the exam until instructed to do so.
2. Students must obey the orders and requests by all proctors, TAs, and lecturers.
3. No student may leave in the first 20 min or in the last 10 min of the exam.
4. Books, notes, calculators, or any electronic devices are not allowed on the exam, and they should not even be in sight in the exam room. Students may not look at anybody else's test, and may not communicate with anybody else except, if they have a question, with their TA or lecturer.
5. After time is called, the students have to put down all writing instruments and remain in their seats, while the TAs will collect the scantrons and the exams.
6. Any violation of these rules and any act of academic dishonesty may result in severe penalties. Additionally, all violators will be reported to the Office of the Dean of Students.

I have read and understand the exam rules stated above:

STUDENT NAME: ____

STUDENT SIGNATURE: ____

1. Find the domain of $g(x) = \dfrac{e^x}{\ln \sqrt[4]{x - 10}}$

A. $(10, \infty)$

B. $[10, \infty)$

C. (e^{11}, ∞)

D. $(10, 11) \cup (11, \infty)$

E. $[10, 11) \cup (11, \infty)$

2. Which of the following has a removable discontinuity at $x = -3$?

A. $f(x) = \dfrac{x^2 - 9}{x - 3}$

B. $f(x) = \dfrac{1}{\sqrt{x + 3}}$

C. $f(x) = \dfrac{x^2 - 9}{x + 3}$

D. $\ln(x + 3)$

E. $\sqrt[3]{x + 3}$

3. If $f(x) = \dfrac{x}{1+2x}$, find $f^{-1}(1)$.

A. 1

B. $\frac{1}{3}$

C. 3

D. $-\frac{1}{3}$

E. -1

4. Choose the right statement which describes ALL the horizontal and vertical asymptotes of the function

$$f(x) = \frac{e^x + 1}{e^x - 1}$$

A. Horizontal Asymptote(s): $y = 1$, $y = -1$, Vertical Asymptote(s): None

B. Horizontal Asymptote(s): $y = 1$, Vertical Asymptote(s): $x = 1$

C. Horizontal Asymptote(s): $y = 1$, Vertical Asymptote(s): $x = 0$

D. Horizontal Asymptote(s): $y = 1$, $y = -1$, Vertical Asymptote(s): $x = 0$

E. Horizontal Asymptote(s): None, Vertical Asymptote(s): $x = 0$

5. Which of the following is TRUE?

I. $\sin^{-1}\left(\sin\left(\frac{2\pi}{3}\right)\right) = \frac{\pi}{3}$

II. $\cos^{-1}\left(\cos\left(\frac{5\pi}{4}\right)\right) = -\frac{\pi}{4}$

III. $\csc\left(\tan^{-1}\left(\frac{1}{x}\right)\right) = \sqrt{x^2+1}$

A. I only

B. II only

C. III only

D. I and II only

E. I and III only

6. Find the derivative of $y = \dfrac{e^x - e^{-x}}{e^x + e^{-x}}$

A. 0

B. 1

C. $\dfrac{2}{(e^x + e^{-x})^2}$

D. $\dfrac{4}{(e^x + e^{-x})^2}$

E. $\dfrac{1}{e^{2x} + e^{-2x}}$

7. If

$$\tan y = \frac{2}{x^2}$$

find y' when $x = -1$

A. 1
B. $\frac{4}{5}$
C. $\frac{-2}{5}$
D. $\frac{2}{5}$
E. $\frac{-4}{5}$

8. Find $f'(1)$ if

$$f(x) = \ln\left(\frac{x}{x^2+2}\right)$$

A. $\frac{-1}{3}$
B. $\frac{-2}{3}$
C. $\frac{1}{3}$
D. $\frac{2}{3}$
E. 1

9. Find the derivative of $y = \cos^2(x^2)$

A. $-\sin^2(x^2)$

B. $4x\sin(x^2)\cos(x^2)$

C. $-4x\sin(x^2)\cos(x^2)$

D. $4x\cos(x^2)$

E. $-4x\sin(x^2)$

10. If $g(t) = f(v)$ where $v = \sin t$, then $g'(t) = ?$

A. $f(\cos t)$

B. $f'(\cos t)$

C. $f'(\sin t) + \cos t$

D. $f'(\sin t)\cos t$

E. $f(\sin t)\cos t$

11. If a certain radioactive substance has a half-life of 7 days, how long, in days, does it take for the sample to decay to $\frac{1}{3}$ of its original amount?

A. $\dfrac{\ln 14}{\ln 2}$

B. $\dfrac{3\ln 7}{\ln 2}$

C. $\dfrac{7\ln 3}{\ln 2}$

D. $\dfrac{3\ln 2}{\ln 7}$

E. $\dfrac{7\ln 2}{\ln 3}$

12. Find the absolute maximum and minimum **values**, on the interval $[0, 1]$, of the function $f(x) = \cos(\pi x) + \sin(\pi x)$.

A. max: $\sqrt{2}$; min: -1

B. max: $\frac{\pi}{4}$; min: π

C. max: 2; min: -2

D. max: 1; min: -1

E. max: π; min: 0

13.

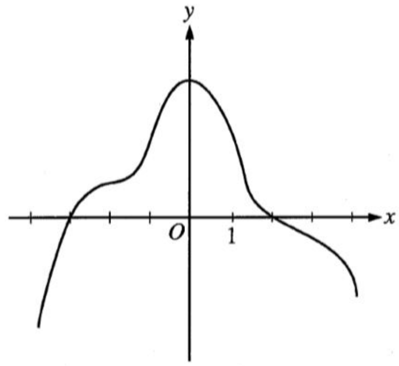
$$\lim_{x\to 0} \frac{\cos(2x)-1}{x^2} =$$

A. 0
B. 1
C. -1
D. 2
E. -2

14. Determine the value of b so that $f(x) = x^2 + \dfrac{b}{x^3}$ has an inflection point at $x = 1$

A. 1
B. $-\dfrac{1}{6}$
C. $\dfrac{1}{3}$
D. $-\dfrac{1}{3}$
E. $\dfrac{1}{6}$

15. The graph of f', **the derivative** of the function f, is shown below. Which of the following statements must be true?



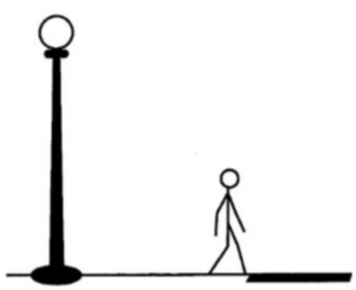
I. f has a relative minimum at $x = -3$

II. The graph of f has an inflection point at $x = -2$

III. The graph of f is concave downward for $0 < x < 4$

A. I only
B. II only
C. III only
D. I and II only
E. I and III only

16. A person whose height is 6 feet is walking away from the base of a streetlight along a straight path at a rate of 4 feet per second. If the height of the streetlight is 15 feet, what is the rate at which the person's shadow is lengthening?



A. $\frac{3}{2}$
B. $\frac{8}{3}$
C. $\frac{15}{4}$
D. 6
E. 10

17. If $f(x) = x^3 - 2x^2 - 3x$, $0 \le x \le 2$, then find a number c that satisfies the conclusion of the Mean Value Theorem.

A. $\frac{3}{4}$

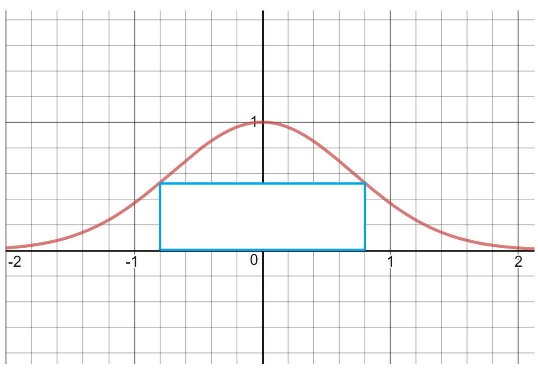
B. 1

C. $\frac{5}{4}$

D. $\frac{4}{3}$

E. $\frac{3}{2}$

18. Find the area of the largest rectangle that can be inscribed under the curve $y = e^{-x^2}$ and above the x-axis.



A. $\sqrt{\frac{2}{e}}$

B. $\sqrt{2e}$

C. $\frac{2}{e}$

D. $\frac{1}{\sqrt{2e}}$

E. $\frac{2}{e^2}$

19. Use a Riemann Sum to **estimate** the area under the graph of $f(x) = \sqrt{x}$ from $x = 1$ to $x = 4$ using **three** approximating rectangles and **left** endpoints.

A. $\sqrt{1} + \sqrt{2} + \sqrt{3}$

B. $\sqrt{2} + \sqrt{3} + \sqrt{4}$

C. $\sqrt{1} + \sqrt{2} + \sqrt{3} + \sqrt{4}$

D. $\frac{1}{3}(\sqrt{1} + \sqrt{2} + \sqrt{3})$

E. $\frac{1}{3}(\sqrt{2} + \sqrt{3} + \sqrt{4})$

20. If $y(x) = \displaystyle\int_3^{\tan(x)} \sqrt{\sqrt{t} + 6t}\, dt$, use Part 1 of the Fundamental Theorem of Calculus to find $y'(\frac{\pi}{4})$.

A. $\sqrt{7}$

B. $2\sqrt{7}$

C. $\sqrt{\frac{3\pi}{2} + \sqrt{\frac{\pi}{4}}}$

D. $2\sqrt{\frac{3\pi}{2} + \sqrt{\frac{\pi}{4}}}$

E. 2

21.

$$\int_1^4 \left(\frac{3}{x} + \frac{3}{x^2}\right)\, dx =$$

A. $3\ln 4 + \dfrac{9}{4}$

B. $3\ln 4 + 3\ln 16$

C. $3\ln 4 - \ln 16$

D. $3\ln 4 - \dfrac{15}{4}$

E. $3\ln 4 - \dfrac{3}{4}$

22. Which of the following statements are correct?

I. Two different functions $f(x)$ and $g(x)$ can have the same derivative

II. An antiderivative of $3x^2$ is $x^3 + \pi^3$

III. If $P(x)$ is an antiderivative of $p(x)$ and $Q(x) = P(x) + 2$, then $Q(x)$ is an antiderivative of $p(x)$

A. None of them

B. I only

C. II only

D. III only

E. All of them

23.

$$\int \frac{x\;dx}{\sqrt{9-x^2}} =$$

A. $-\frac{1}{2}\ln\sqrt{9-x^2}+C$

B. $\sin^{-1}\frac{x}{3}+C$

C. $-\sqrt{9-x^2}+C$

D. $-\frac{1}{4}\sqrt{9-x^2}+C$

E. $2\sqrt{9-x^2}+C$

24.

$$\int_{\pi/6}^{\pi/2} \cot x\;dx$$

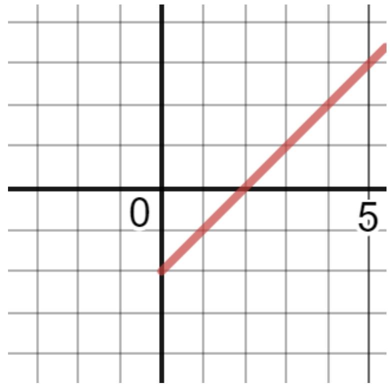
A. $\ln\frac{1}{2}$

B. $-\ln\frac{1}{2}$

C. $-\ln(2-\sqrt{3})$

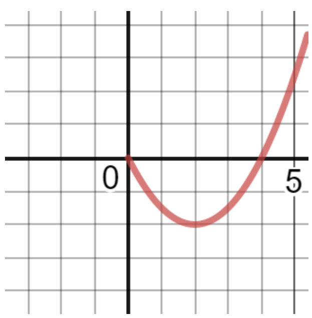
D. $\ln(\sqrt{3}-1)$

E. $\ln\frac{1}{4}$

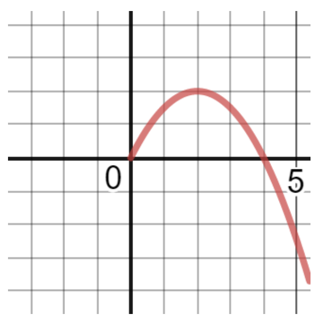


25. The figure above shows the graph of $f(x)$. Which of the graphs below is $\int_0^x f(t)\,dt$?

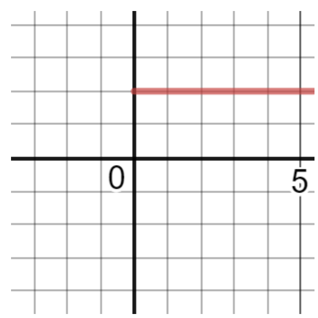
A



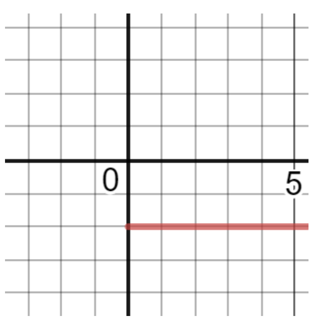
B



C



D



E