

Name: \_\_\_\_\_ PUID: \_\_\_\_\_

Your TA's Name: \_\_\_\_\_ Recitation Time: \_\_\_\_\_

MA 16100

SPRING 2024

**FINAL EXAM VERSION A**

The problems are numbered 1-24. Indicate your answer by filling in the appropriate circle next to the answer choice. Partial credit will **not** be awarded.

This exam is out of 100 points. Each problem is worth 4 points each. You will receive 4 points for signing the bottom of this page.

Extra scratch paper is not permitted. Write all your work in this exam booklet.

Write your name and PUID on each page. This will help us locate and successfully grade your test if the pages become separated.

You may not leave the room before 20 minutes have passed. If you finish the exam between when 20 and 110 minutes have passed, you may leave the room after turning in the exam booklet. If you finish within the last 10 minutes of the exam, you **MUST REMAIN SEATED** until your TA comes and collects your exam booklet.

**Exam Policies:**

1. Students may not open the exam until instructed to do so.
2. Students must obey the orders and requests by all proctors, TAs, and the instructor.
3. No student may leave in the first 20 min or in the last 10 min of the exam.
4. Books, notes, calculators, or any electronic devices are not allowed on the exam, and they should not even be in sight in the exam room. Students may not look at anybody else's test, and may not communicate with anybody else except, if they have a question, with their TA or the instructor.
5. After time is called, the students have to put down all writing instruments and remain in their seats, while the TAs will collect the exams.
6. Any violation of these rules and any act of academic dishonesty may result in severe penalties. Additionally, all violators will be reported to the Office of the Dean of Students.

I have read and understand the exam rules stated above:

**Student Signature:** \_\_\_\_\_

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1. (4 Points) If  $\log_8(x) = 3$  and  $\log_4(y) = 2$  what is  $\log_2\left(\frac{y^2}{x}\right)$ ?

- A 5.
- B 3.
- C -1.
- D 4.
- E -3.

2. (4 Points) What is the domain of the function  $f(x) = \frac{3x}{4 - \sqrt{x}}$ ?

- A  $[0, 4) \cup (4, \infty)$ .
- B  $[0, 16) \cup (16, \infty)$ .
- C  $(-\infty, -4) \cup (-4, 0]$ .
- D  $(-\infty, 4) \cup (4, \infty)$ .
- E  $(-\infty, -16) \cup (-16, 16) \cup (16, \infty)$ .

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3. (4 Points) For which  $x$  in  $[0, \frac{2\pi}{3}]$  do we have  $\sin(3x) = -\cos(3x)$ ?

- A  $x = \frac{\pi}{4}, \frac{7\pi}{12}$ .
- B  $x = \frac{3\pi}{4}, \frac{7\pi}{4}$ .
- C  $x = \frac{\pi}{4}, \frac{5\pi}{12}$ .
- D  $x = \frac{\pi}{12}, \frac{7\pi}{12}$ .
- E  $x = \frac{\pi}{12}, \frac{5\pi}{12}$ .

4. (4 Points) Compute  $\lim_{x \rightarrow 5^+} \frac{x+1}{x^2-25}$ .

- A 0.
- B  $-\infty$ .
- C  $\frac{3}{5}$ .
- D  $\infty$ .
- E  $\frac{1}{10}$ .

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5. (4 Points) Compute  $\lim_{x \rightarrow \infty} \frac{x \arctan(x)}{x + 1}$ .

- A 1.
- B  $\frac{\pi}{2}$ .
- C  $\frac{\pi}{4}$ .
- D 0.
- E  $\infty$ .

6. (4 Points) Compute  $\lim_{x \rightarrow 7^+} \frac{x^2 - 5x - 14}{x^3 - 49x}$ .

- A  $\infty$ .
- B  $\frac{9}{7}$ .
- C 0.
- D  $\frac{9}{98}$ .
- E  $-\infty$ .

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7. (4 Points) Where is function

$$f(x) = \begin{cases} x^2 & \text{if } x \leq -2 \\ 2 - x & \text{if } -2 < x < 1 \\ x^2 & \text{if } 1 \leq x < 4 \\ 4x & \text{if } 4 \leq x \end{cases}$$

continuous?

- A** It is continuous for all  $x$  except for  $x = 1$ .
- B** It is continuous for all  $x$ .
- C** It is continuous for all  $x$  except for  $x = -2$ .
- D** It is continuous for all  $x$  except for  $x = -2$ ,  $x = 1$  and  $x = 4$ .
- E** It is continuous for all  $x$  except for  $x = 4$ .

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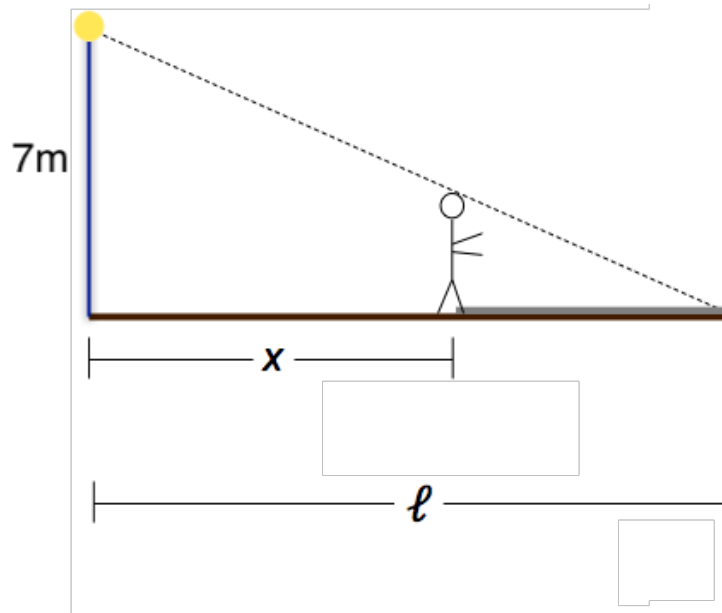
8. (4 Points) Find all asymptotes of the function

$$f(x) = \frac{x^4 + x^3 - 25x^2 - 25x + 1}{x^3 - 25x}$$

- A** Vertical asymptotes  $x = 0$ , and  $x = 5$ ; slant asymptote  $y = x + 1$ .
- B** Vertical asymptotes  $x = -5$ ,  $x = 0$ , and  $x = 5$ ; slant asymptote  $y = x$ .
- C** Vertical asymptotes  $x = -5$ ,  $x = 0$ , and  $x = 5$ ; horizontal asymptote  $y = 1$ .
- D** Vertical asymptotes  $x = -5$ ,  $x = 0$ , and  $x = 5$ ; slant asymptote  $y = x + 1$ .
- E** Vertical asymptotes  $x = 0$ , and  $x = 5$ ; horizontal asymptote  $y = 0$ .

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9. (4 Points) A person walks away from a building which has a light which is 7 metres up the wall. If the person is 1.5 metres tall and is walking away from the building at a speed of 1 metre per second, how fast is their shadow moving when they are 20 metres away from the building?



- A  $\frac{11}{14}$  metres per second.
- B  $\frac{11}{5}$  metres per second.
- C  $\frac{5}{11}$  metres per second.
- D  $\frac{5}{2}$  metres per second.
- E  $\frac{14}{11}$  metres per second.

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10. (4 Points) Compute  $\frac{dy}{dx}$  for the curve

$$y^2 = x^3 - x + 1$$

at the point (1, 1).

- A 0.
- B -1.
- C 1.
- D -2.
- E 3.



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11. (4 Points) A ball is thrown upward and its height is modelled by the function

$$p(t) = -4.9t^2 + 9.8t + 5.1.$$

What is the maximum height that the ball obtains?

- A 10m.
  - B 12m.
  - C 0.2m.
  - D 5.1m.
  - E 14.9m.
12. (4 Points) Determine when the function  $f(x) = \frac{x}{x^2 - 3x + 2}$  is increasing.

- A  $(-\sqrt{2}, \sqrt{2})$ .
- B  $(-\infty, -\sqrt{2}) \cup (\sqrt{2}, \infty)$ .
- C  $(-\infty, -\sqrt{2}) \cup (\sqrt{2}, 2) \cup (2, \infty)$ .
- D  $(-\infty, 1) \cup (1, 2) \cup (2, \infty)$ .
- E  $(-\sqrt{2}, 1) \cup (1, \sqrt{2})$ .

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13. (4 Points) Determine when the function

$$f(x) = x^4 - 3x^3 + 3x^2 - 7x + 1$$

is concave down.

- A  $(-3, 4)$ .
- B  $\left(-\infty, \frac{1}{2}\right) \cup (1, \infty)$ .
- C  $(-\infty, -3) \cup (4, \infty)$ .
- D  $\left(\frac{1}{2}, 1\right)$ .
- E  $(1, \infty)$ .

14. (4 Points) If  $f(x)$  is differentiable for all real numbers and the derivative  $f'(x)$  of  $f(x)$  is given by the formula

$$f'(x) = x^3(x - 2)^5(x - 4)(x^2 + 1).$$

How many local minima does  $f(x)$  have?

- A 0.
- B 1.
- C 2.
- D 3.
- E 4.

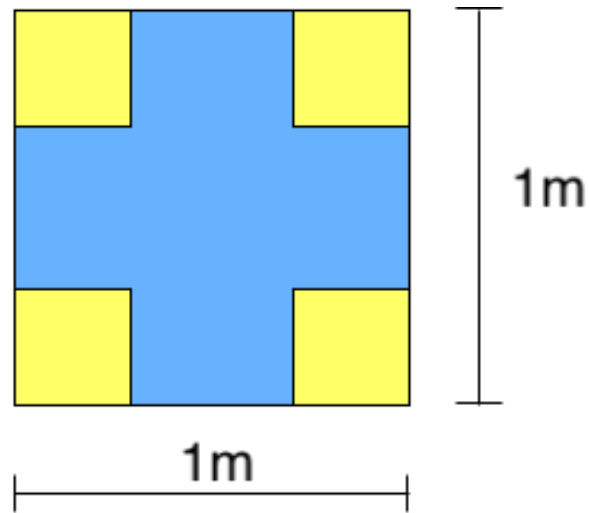
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15. (4 Points) What is the largest possible product of two non-negative numbers  $a$  and  $b$  whose sum is 10?

- A 40.
- B 24.
- C 0.
- D 10.
- E 25.

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16. (4 Points) Four squares of equal size are to be cut from each of the corners of a one metre by one metre piece of cardboard. Once the corners are cut, the newly-formed flaps are to be folded up in order to create a lidless box. What is the maximum volume of an open-topped box created in this way?



- A  $\frac{1}{27}$ .
- B  $\frac{2}{27}$ .
- C 0.
- D  $\frac{9}{128}$ .
- E  $\frac{1}{16}$ .

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17. (4 Points) Compute  $\lim_{x \rightarrow 0^+} (1 + x)^{\frac{1}{\sin(x)}}$ .

- A 1.
- B  $e$ .
- C 0.
- D  $\infty$ .
- E  $e^{-1}$ .

18. (4 Points) Determine which of the following functions is an antiderivative of

$$f(x) = 5x^4 + \ln(7)7^x + \cos(x).$$

- A  $F(x) = x^5 + 7^x + \sin(x) - 3$ .
- B  $F(x) = x^5 + \ln(7)7^x + \sin(x)$ .
- C  $F(x) = 5x^4 + 7^x + \sin(x) + 1$ .
- D  $F(x) = x^5 + 7^x + \cos(x) + 5$ .
- E  $F(x) = x^5 + 7^x + \arcsin(x) - 2$ .

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19. (4 Points) Determine the average value of the function  $f(x) = x^2 + x + 1$  on the interval  $[0, 3]$ .

- A 11.
- B  $\frac{33}{2}$ .
- C  $\frac{22}{3}$ .
- D  $\frac{11}{2}$ .
- E 22.

20. (4 Points) Find the value of the left endpoint Riemann sum for the function  $y = x^2$  on the interval  $[0, 2]$  partitioned into 4 pieces.

- A  $\frac{7}{2}$ .
- B  $\frac{11}{2}$ .
- C  $\frac{13}{4}$ .
- D  $\frac{7}{4}$ .
- E  $\frac{5}{4}$ .

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21. (4 Points) Compute  $\int_0^{\frac{3\pi}{2}} \sin(x) dx$ .

- A 1.
- B 3.
- C  $\pi$ .
- D  $\frac{3\pi}{2}$ .
- E 0.

22. (4 Points) The value of  $\int_{-4}^4 \sqrt{16 - x^2} dx$  is  $8\pi$ , the area of a half-circle of radius 4. Use this fact to compute

$$\int_{-4}^4 \left( -\pi + 3\sqrt{16 - x^2} \right) dx$$

- A 0.
- B  $24\pi - 8$ .
- C  $8 - 24\pi$ .
- D  $8\pi$ .
- E  $16\pi$ .

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23. (4 Points) Compute  $\int (\sin^2(x) + \sin(x) + 1) \cos(x) dx$ .

- A  $\sin^2(x) + \sin(x) + C$ .
- B  $\frac{\sin^3(x)}{3} + \frac{\sin^2(x)}{2} + \sin(x) + C$ .
- C  $\frac{x^3}{3} + \frac{x^2}{2} + x + C$ .
- D  $\sin(x^2 + x + 1) + C$ .
- E  $\frac{\sin^3(x) \cos(x)}{3} + \frac{\sin^2(x) \cos(x)}{2} + \sin(x) \cos(x) + C$ .

24. (4 Points) Compute  $\int_0^2 \left(x^2 - \frac{3}{2}x\right)^{43} \left(2x - \frac{3}{2}\right) dx$

- A  $\frac{1}{43}$ .
- B  $\frac{2^{43} - 1}{43}$ .
- C 0.
- D  $\frac{2^{44} - 1}{44}$ .
- E  $\frac{1}{44}$ .



*(This page may be used for scratch work.)*

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