

Name: _____

10-digit PUID: _____

Lecturer: _____

Recitation Instructor: _____

Recitation Time: _____

Instructions:

1. This package contains 14 problems worth 7 points each.
2. Please supply all information requested. You get 2 points for supplying all information correctly.
3. Work only in the space provided, or on the backside of the pages. Circle your choice for each problem in this booklet.
4. No books, notes, calculator or any electronic devices, please.

1. D 2. B 3. A 4. D 5. C 6. D

7. A 8. E 9. A 10. E

11. E 12. C 13. B

14. C

1. $\lim_{n \rightarrow \infty} \frac{3n + 1}{n^2 + 4(-1)^n} =$

- A. $1/4$
- B. $3/4$
- C. 3
- D. 0
- E. The limit does not exist

2. Which among the following series converges?

I. $\sum_{i=1}^{\infty} \frac{2i + 1}{3i + 2};$

II. $\sum_{j=1}^{\infty} \frac{1}{6j + 1};$

III. $\sum_{k=1}^{\infty} \frac{1}{\sqrt{k}}$

- A. All three
- B. Neither
- C. Only I and III
- D. Only II
- E. Only I

3. Evaluate

$$5 - \frac{10}{3} + \frac{20}{9} - \frac{40}{27} + \frac{80}{81} - \frac{160}{243} + \dots$$

- A. 3
- B. $5/3$
- C. 15
- D. 10
- E. $10/3$

4. For what values of p does the series

$$\sum_{n=1}^{\infty} \sqrt{\frac{3}{n^p + 1}}$$

converge?

- A. $p \geq 1$
- B. $p > 1$
- C. $p \geq 2$
- D. $p > 2$
- E. $p > 0$

5. Which is true? The series $\sum_{m=1}^{\infty} \frac{2^{-m}}{\sqrt{m} + 3}$
- A. converges by comparison with $\sum_{m=1}^{\infty} 1/\sqrt{m}$.
 - B. diverges by comparison with $\sum_{m=1}^{\infty} 1/\sqrt{m}$.
 - C. converges by comparison with $\sum_{m=1}^{\infty} 2^{-m}$.
 - D. diverges by comparison with $\sum_{m=1}^{\infty} 2^{-m}$.
 - E. The comparison test is not applicable.
6. Which statement is false?
- A. If $\{a_n\}$ is a bounded, increasing sequence, then it is convergent.
 - B. If $\sum_{n=1}^{\infty} b_n$ is convergent, then $\lim_{n \rightarrow \infty} b_n = 0$.
 - C. $\sum_{n=1}^{\infty} r^n$ diverges when $|r| \geq 1$.
 - D. If $\sum_{n=1}^{\infty} b_n$ is divergent and if $0 \leq a_n \leq b_n$, then $\sum_{n=1}^{\infty} a_n$ must be divergent.
 - E. If $a_n > 0$, $b_n > 0$, $\lim_{n \rightarrow \infty} a_n/b_n = L$ is finite and positive, and if $\sum_{n=1}^{\infty} b_n$ is divergent, then $\sum_{n=1}^{\infty} a_n$ must be divergent.

7. For the series $\sum_{k=1}^{\infty} (-1)^k k$, the partial sum s_5 equals

- A. -3
- B. -2
- C. 3
- D. 5
- E. -5

8. For the series below, which statement is true?

I. $\sum_{j=1}^{\infty} (-1)^j$;

II. $\sum_{k=1}^{\infty} \frac{(-1)^k}{\sqrt{k}}$;

III. $\sum_{m=0}^{\infty} (-1)^m e^{-m}$.

- A. All are conditionally convergent, none of them is absolutely convergent.
- B. All are conditionally convergent, III is also absolutely convergent.
- C. None of them is conditionally convergent, III is absolutely convergent.
- D. None of them is absolutely convergent, II and III are conditionally convergent.
- E. II is conditionally convergent, III is absolutely convergent.

9. The set of x for which the series $\sum_{k=1}^{\infty} e^{-kx}/k!$ converges is

- A. all x
- B. $x \leq 1$
- C. $x < 1$
- D. $x \geq 0$
- E. $x > e$

10. The series $\sum_{n=0}^{\infty} \frac{(2n+1)^n}{(n+1)^{2n}}$

- A. diverges by the alternating series test.
- B. diverges by the integral test.
- C. converges by comparison with $\sum 2/n^n$.
- D. diverges by the ratio test.
- E. converges by the root test.

11. The radius of convergence of the series $\sum_{n=1}^{\infty} 2^n x^n / (n+1)$ is
- A. 0
 - B. ∞
 - C. 1
 - D. 2
 - E. 1/2
12. Given that the power series $\sum_{m=1}^{\infty} (x-1)^m / \sqrt[m]{m}$ has radius of convergence 1, its interval of convergence is
- A. $[0, 2]$
 - B. $(0, 2]$
 - C. $[0, 2)$
 - D. $(0, 2)$
 - E. none of the above

13.
$$\int_0^x \frac{t dt}{1-t^3} =$$

A.
$$\sum_{n=0}^{\infty} x^{3n+1}$$

B.
$$\sum_{n=0}^{\infty} \frac{x^{3n+2}}{3n+2}$$

C.
$$\sum_{n=0}^{\infty} nx^{3n}$$

D.
$$\sum_{n=0}^{\infty} (3n+1)x^{3n}$$

E.
$$\sum_{n=0}^{\infty} 3nx^{3n-1}$$

14. Starting with the power series of $1/(1+2x)$, compute the power series that represents $1/(1+2x)^2$.

A.
$$\sum_{m=0}^{\infty} (2x)^{2m}$$

B.
$$\sum_{m=0}^{\infty} 2m(2x)^m$$

C.
$$\sum_{m=1}^{\infty} m(-2)^{m-1}x^{m-1}$$

D.
$$\sum_{m=1}^{\infty} m^2 2^{-m} x^{2m-1}$$

E.
$$\sum_{m=1}^{\infty} (-1)^m 2^{2m+1} x^{2m+1}$$