MA 16200 Exam III, Nov 16, 2017

Name ____

10–digit PUID number ______________________________________

Recitation Instructor ______________________________________

Recitation Section Number and Time ______________________________

Instructions: **MARK TEST NUMBER 68 ON YOUR SCANTRON**

1. Do not open this booklet until you are instructed to.
2. Fill in all the information requested above and on the scantron sheet. On the scantron sheet fill in the little circles for your name, section number and PUID.
3. This booklet contains 12 problems, each worth 8 points. You will get 4 points for correctly supplying information above and on the scantron.
4. For each problem mark your answer on the scantron sheet and also **circle it in this booklet**.
5. Work only on the pages of this booklet.
6. Books, notes, calculators or any electronic device are not allowed during this test and they should not even be in sight in the exam room. You may not look at anybody else's test, and you may not communicate with anybody else, except, if you have a question, with your instructor.
7. You are not allowed to leave during the first 20 and the last 10 minutes of the exam.
8. When time is called at the end of the exam, put down your writing instruments and remain seated. The TAs will collect the scantrons and the booklets.

Questions

1.

$$I.\ \sum_{n=1}^{\infty}\frac{3^n}{n^4};\quad II.\ \sum_{n=1}^{\infty}(-1)^{n-1}\frac{1}{\ln^2(n+1)};\quad III.\ \sum_{n=1}^{\infty}(-1)^{n-1}\frac{\sqrt{n}}{n+1}.$$

Which of the following statement is true?

A. Only II is convergent

B. II and III are convergent

C. All three are divergent

D. I and II are convergent

E. All three are convergent

2. Which of the following statement is false?

A. $\sum_{n=1}^{\infty} r^n$ diverges for all $|r| \geq 1$.

B. $\sum_{n=1}^{\infty}(-1)^{n-1}\frac{1}{\sqrt{n}}$ is absolutely convergent.

C. $\sum_{n=1}^{\infty}\frac{2^n}{n^p}$ diverges for all $p > 1$.

D. $\sum_{n=1}^{\infty}\frac{1}{n^p+n}$ is convergent for all $p > 1$.

E. $\sum_{n=1}^{\infty}\frac{n^2}{r^n}$ converges for all $r > 1$.

3. The length of the curve

$$x = t^4/2, \quad y = (1+t^5)/5, \quad 0 \le t \le 1$$

is given by

A. $\displaystyle\int_0^1 \sqrt{\frac{t^8}{4} + \frac{(1+t^5)^2}{25}}\,dt$

B. $\displaystyle\int_0^1 \sqrt{\frac{t^4(1+t^5)}{10}}\,dt$

C. $\displaystyle\int_0^1 t^3\sqrt{4+t^2}\,dt$

D. $\displaystyle\int_0^1 t^2\sqrt{4-t^2}\,dt$

E. $\displaystyle\int_0^1 \sqrt{\frac{t^4}{2} + \frac{1+t^5}{5}}\,dt$

4. The first three terms of the binomial series for $f(x) = (1+2x)^{-1/4}$ are

A. $1 - \frac{1}{4}x + \frac{5}{32}x^2$

B. $1 - \frac{1}{2}x + \frac{5}{8}x^2$

C. $1 - \frac{1}{5}x + \frac{5}{9}x^2$

D. $1 - \frac{1}{3}x + \frac{5}{9}x^2$

E. $1 - \frac{1}{2}x + \frac{3}{25}x^2$

5. Find the radius and interval of convergence of the power series

$$\sum_{n=1}^{\infty} (-1)^{n-1} \frac{1}{n} (x+2)^n.$$

A. radius of convergence: 1; interval of convergence: (-3,-1)

B. radius of convergence: 1; interval of convergence: (-3,-1]

C. radius of convergence: 1; interval of convergence: [-3,-1)

D. radius of convergence: ∞; interval of convergence: $(-\infty, \infty)$

E. radius of convergence: 2; interval of convergence: (-4,0)

6. Find the power series for $\dfrac{x}{3+x}$.

A. $\sum_{n=0}^{\infty} (-1)^n 3^{-n-1} x^{n+1}$

B. $\sum_{n=0}^{\infty} 3^{-n} x^n$

C. $\sum_{n=0}^{\infty} 3^{-n-1} x^n$

D. $\sum_{n=0}^{\infty} (-1)^n 3^{-n+1} x^{n+1}$

E. $\sum_{n=0}^{\infty} (-1)^n 3^{-n} x^{n+1}$

7. The Maclaurin series of $\dfrac{x}{\sqrt{1-x^2}}$ is

A. $\displaystyle\sum_{k=0}^{\infty} \frac{2 \cdot 4 \cdots (2k)}{2^k k!} x^{2k}$

B. $\displaystyle\sum_{k=0}^{\infty} \frac{1 \cdot 3 \cdots (2k-1)}{2^k k!} x^{2k+1}$

C. $\displaystyle\sum_{k=0}^{\infty} (-1)^k \frac{1 \cdot 3 \cdots (2k-1)}{k!} x^{2k-1}$

D. $\displaystyle\sum_{k=0}^{\infty} (-1)^k \frac{1 \cdot 3 \cdots (2k-1)}{k!} x^{2k}$

E. $\displaystyle\sum_{k=0}^{\infty} (-1)^{k+1} \frac{1 \cdot 3 \cdots (2k+1)}{2^{k+1} k!} x^{2k}$

8. Consider $S = \sum_{m=1}^{\infty} (-1)^{m-1} \frac{1}{m^3}$ and its partial sums S_n. Which of the following is true?

$$|S - S_n| \leq 8 \times 10^{-6} \quad \text{if} \quad I.\ n = 49; \quad II.\ n = 51; \quad III.\ n = 99.$$

A. Only I

B. Only II

C. Only II and III

D. I, II and III

E. Only III

9. In the power series for $\dfrac{1}{x^2}$ about -1, what is the coefficient of $(x+1)^4$?

A. 1

B. 2

C. 3

D. 4

E. 5

10. The equation of the tangent drawn to the curve

$$x = t\cos t, \quad y = t^2 \sin 2t$$

corresponding to $t = \pi$ is

A. $y = \pi x$

B. $y = \pi^2 x + 2\pi$

C. $y = -2x + \pi^2$

D. $y = 2(x + \pi)$

E. $y = -2\pi^2(x + \pi)$

11. Which of the following series converge?

$$I.\ \sum_{k=1}^{\infty}(-1)^{k-1}\sin k; \quad II.\ \sum_{k=1}^{\infty}(-1)^{k-1}\frac{1}{\sqrt{k}+1}; \quad III.\ \sum_{k=1}^{\infty}\frac{\cos k}{k^2+1}$$

A. All three

B. I and II

C. Only III

D. None

E. II and III

12. The Maclaurin series of $x\cos x + \sin 2x$ is

A. $\displaystyle\sum_{n=0}^{\infty}(-1)^n\frac{2n+1+2^{2n+1}}{(2n+1)!}x^{2n+1}$

B. $\displaystyle\sum_{n=0}^{\infty}(-1)^n\frac{2n+2^{2n}}{(2n)!}x^{2n}$

C. $\displaystyle\sum_{n=0}^{\infty}(-1)^n\frac{n+2^{n}}{(n)!}x^{n}$

D. $\displaystyle\sum_{n=0}^{\infty}\frac{n+2^{n}}{(n)!}x^{n}$

E. $\displaystyle\sum_{n=0}^{\infty}\frac{2n+2^{2n+1}}{(2n+1)!}x^{2n+2}$