MA 16200
EXAM 3
11/16/2021
TEST/QUIZ NUMBER: **1010**

NAME ______________________ YOUR TA'S NAME ______________________

STUDENT ID # ____________________ RECITATION TIME ____________________

You must use a #2 pencil on the scantron sheet. Write **1010** in the TEST/QUIZ NUMBER boxes and blacken in the appropriate digits below the boxes. On the scantron sheet, fill in your **TA**'s name for the INSTRUCTOR and **MA 162** for the COURSE number. Fill in whatever fits for your first and last NAME. The STUDENT IDENTIFICATION NUMBER has ten boxes, so use **00** in the first two boxes and your PUID in the remaining eight boxes. Fill in your three-digit SECTION NUMBER. If you do not know your section number, ask your TA. Complete the signature line.
There are **12** questions, each worth 8 points (you will automatically earn 4 points for taking the exam). Blacken in your choice of the correct answer in the spaces provided for questions 1–12. Do all your work in this exam booklet and indicate your answers in the booklet in case the scantron is lost. Use the back of the test pages for scrap paper. Turn in both the scantron sheet and the exam booklet when you are finished.
If you finish the exam before 7:20, you may leave the room after turning in the scantron sheet and the exam booklet. You may not leave the room before 6:50. If you don't finish before 7:20, you MUST REMAIN SEATED until your TA comes and collects your scantron sheet and your exam booklet.

## EXAM POLICIES

1. Students may not open the exam until instructed to do so.
2. Students must obey the orders and requests by all proctors, TAs, and lecturers.
3. No student may leave in the first 20 min or in the last 10 min of the exam.
4. Books, notes, calculators, or any electronic devices are not allowed on the exam, and they should not even be in sight in the exam room. Students may not look at anybody else's test, and may not communicate with anybody else except, if they have a question, with their TA or lecturer.
5. After time is called, the students have to put down all writing instruments and remain in their seats, while the TAs will collect the scantrons and the exams.
6. Any violation of these rules and any act of academic dishonesty may result in severe penalties. Additionally, all violators will be reported to the Office of the Dean of Students.

I have read and understand the exam rules stated above:

STUDENT NAME: ______________________________________

STUDENT SIGNATURE: __________________________________

**1.** Suppose $p_3(x)$ is the Taylor polynomial of order 3 centered at $a = 1$ for the function

$$f(x) = 1/x.$$

Compute $p_3(2)$.

A. 3/8
B. 2
C. 1
D. 1/2
E. 0
F. 9/16

**2.** Describe the following two series: $$\sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{\sqrt{k}} \qquad \text{and} \qquad \sum_{k=2}^{\infty} \frac{(-1)^{k} \ln k}{k}$$

A. Both series converge absolutely.
B. Both series diverge.
C. One series converges conditionally, and one series converges absolutely.
D. One series diverges, and one series converges absolutely.
E. One series diverges, and one series converges conditionally.
F. Both series converge conditionally.

**3.** Two of these series converge, and two diverge. Which two **converge**?

(I) $\sum_{k=1}^{\infty} \frac{1}{\sqrt[5]{k^4+1}}$ (II) $\sum_{k=1}^{\infty} k^{-e}$ (III) $\sum_{k=2}^{\infty} \frac{1}{k(\ln k)^2}$ (IV) $\sum_{k=1}^{\infty} (1+1/k)^k$

A. (II) and (IV)
B. (I) and (II)
C. (I) and (III)
D. (I) and (IV)
E. (III) and (IV)
F. (II) and (III)

**4.** Using the Taylor polynomial of order 2 centered at $a = \frac{\pi}{6}$,

$$\cos\left(\frac{\pi}{5}\right) \approx \frac{\sqrt{3}}{2} - \frac{1}{2}\left(\frac{\pi}{30}\right) - \frac{\sqrt{3}}{4}\left(\frac{\pi}{30}\right)^2.$$

According to the Taylor Remainder Theorem, this approximation has an error equal to

A. $\frac{\cos(c)}{6} \cdot \left(\frac{\pi}{30}\right)^3$ for some $c$ between $\frac{\pi}{6}$ and $\frac{\pi}{5}$
B. $\frac{\sqrt{3}\sin(c)}{12} \cdot \left(\frac{\pi}{30}\right)^3$ for some $c$ between $\frac{\pi}{6}$ and $\frac{\pi}{5}$
C. $\frac{\sqrt{3}\cos(c)}{12} \cdot \left(\frac{\pi}{30}\right)^3$ for some $c$ between $\frac{\pi}{6}$ and $\frac{\pi}{5}$
D. $\frac{\sin(c)}{12} \cdot \left(\frac{\pi}{30}\right)^3$ for some $c$ between $\frac{\pi}{6}$ and $\frac{\pi}{5}$
E. $\frac{\cos(c)}{12} \cdot \left(\frac{\pi}{30}\right)^3$ for some $c$ between $\frac{\pi}{6}$ and $\frac{\pi}{5}$
F. $\frac{\sin(c)}{6} \cdot \left(\frac{\pi}{30}\right)^3$ for some $c$ between $\frac{\pi}{6}$ and $\frac{\pi}{5}$

**5.** The Ratio Test for the series

$$\sum_{k=1}^{\infty} \frac{(k!)^2}{(2k)!}$$

gives a resulting limit of

A. $r = 1$ and the series diverges.

B. $r < 1$ and therefore the series converges.

C. $r > 1$ and therefore the series converges.

D. $r < 1$ and therefore the series diverges.

E. $r > 1$ and therefore the series diverges.

F. $r = 1$ and the series converges.

**6.** Determine the radius of convergence for the power series

$$\sum_{k=1}^{\infty} \frac{2(3x+2)^k}{k}$$

A. 3/4

B. 4/3

C. 3/2

D. 1/3

E. 2/3

F. 3

**7.** The sequence $\left\{\tan\left(\frac{1}{k}\right) - \sec\left(\frac{k}{k^2+1}\right)\right\}$

A. converges to $-1$.

B. converges to 2.

C. diverges.

D. converges to 0.

E. converges to $-2$.

F. converges to 1.

**8.** Which of the following is closest to the value of

$$\sum_{k=1}^{19}\left(-\frac{2}{3}\right)^k \quad ?$$

Hint: $\left(\frac{2}{3}\right)^{20}$ is approximately 0.0003.

A. 0.5991

B. $-0.3991$

C. $-0.4002$

D. $-0.4011$

E. 0.6002

F. 0.6011

**9.** Two of the following statements are true, and two are false. Which two are **true**?

(I) If the sequence $a_1, a_2, a_3, \ldots$ converges, then the series $a_1 + a_2 + a_3 + \ldots$ converges.
(II) If the sequence $a_1, a_2, a_3, \ldots$ diverges, then the series $a_1 + a_2 + a_3 + \ldots$ diverges.
(III) If the series $a_1 + a_2 + a_3 + \ldots$ converges, then the sequence $a_1, a_2, a_3, \ldots$ converges.
(IV) If the series $a_1 + a_2 + a_3 + \ldots$ diverges, then the sequence $a_1, a_2, a_3, \ldots$ diverges.

A. (I) and (IV)

B. (III) and (IV)

C. (I) and (III)

D. (II) and (III)

E. (II) and (IV)

F. (I) and (II)

**10.** Find a formula for the $n$th partial sum,

$$S_n = \sum_{k=1}^{n} \frac{1}{k^2 + 5k + 6}$$

Hint: partial fractions

A. $S_n = \frac{5}{12} - \frac{1}{n+2}$

B. $S_n = \frac{1}{30} + \frac{n}{20}$

C. $S_n = \frac{1}{4} - \frac{1}{n+2}$

D. $S_n = \frac{1}{12} + \frac{1}{n+2} - \frac{1}{n+3}$

E. $S_n = \frac{1}{3} - \frac{1}{n+3}$

F. $S_n = \frac{1}{n+2} - \frac{1}{n+3}$

**11.** Two of these series converge, and two diverge. Which two **converge**?

(I) $\displaystyle\sum_{k=1}^{\infty} \frac{e^k}{e^{2k}+1}$ (II) $\displaystyle\sum_{k=2}^{\infty} \frac{(-1)^k(k+1)}{k}$ (III) $\displaystyle\sum_{k=1}^{\infty} \frac{\pi^k}{3^k-2}$ (IV) $\displaystyle\sum_{k=2}^{\infty} \frac{1}{(\ln k)^k}$

A. (I) and (II)
B. (I) and (III)
C. (I) and (IV)
D. (II) and (IV)
E. (III) and (IV)
F. (II) and (III)

**12.** Use a limit comparison test to draw an appropriate conclusion about

$$\sum_{k=1}^{\infty} \frac{1+k}{\sqrt{3+3k+k^3}}$$

A. The series converges by considering comparison series $\sum \frac{1}{k^2}$
B. The series converges by considering comparison series $\sum \frac{1}{k^{1/2}}$
C. The series diverges by considering comparison series $\sum \frac{1}{k^{1/2}}$
D. The series diverges by considering comparison series $\sum \frac{1}{k^{3/2}}$
E. The series diverges by considering comparison series $\sum \frac{1}{k^2}$
F. The series converges by considering comparison series $\sum \frac{1}{k^{3/2}}$