MA 16200
EXAM 3 Form 01
November 15, 2022

NAME ___________________ YOUR TA'S NAME _____________________

STUDENT ID # ____________ RECITATION TIME _________________

Be sure the paper you are looking at right now is GREEN! Write the following in the TEST/QUIZ NUMBER boxes and blacken in the appropriate spaces below the boxes on the scantron: 01

You must use a #2 pencil on the scantron answer sheet. Fill in the following on your scantron and blacken the bubbles:

1. Your name. If there aren't enough space for your name, fill in as much as you can.
2. Section number. If you don't know your section number, ask your TA.
3. Test/Quiz number: 01
4. Student Identification Number: This is your Purdue ID number with two leading zeros.

There are 12 questions, each worth 8 points (you will automatically earn 4 points for filling out your student ID number correctly). Blacken in your choice of the correct answer in the spaces provided for questions 1-12. Do all your work in this exam booklet. Use the back of the test pages for scrap paper. Turn in both the scantron and the exam booklet when you are finished.

If you finish the exam before 8:50, you may leave the room after turning in the scantron sheet and the exam booklet. You may not leave the room before 8:20. If you don't finish before 8:50, you MUST REMAIN SEATED until your TA comes and collects your scantron sheet and your exam booklet.

EXAM POLICIES

1. Students may not open the exam until instructed to do so.
2. Students must obey the orders and requests by all proctors, TAs, and lecturers.
3. No student may leave in the first 20 min or in the last 10 min of the exam.
4. Books, notes, calculators, or any electronic devices are not allowed on the exam, and they should not even be in sight in the exam room. Students may not look at anybody else's test, and may not communicate with anybody else except, if they have a question, with their TA or lecturer.
5. After time is called, the students have to put down all writing instruments and remain in their seats, while the TAs will collect the scantrons and the exams.
6. Any violation of these rules and any act of academic dishonesty may result in severe penalties. Additionally, all violators will be reported to the Office of the Dean of Students.

I have read and understand the exam rules stated above:

STUDENT SIGNATURE: ___________________
1. Consider the two series
\[ \sum_{n=0}^{\infty} \frac{3n^2 - 1}{n^5 + 1}, \quad \sum_{n=0}^{\infty} \frac{1}{\sqrt{n^2 + 2}}. \]

A. Both are convergent  
B. Both are divergent  
C. The first one is convergent while the second one is divergent  
D. The first one is divergent while the second one is convergent  
E. None of the above

2. Consider the two sequences
\[ a_n = \frac{1}{n} - \frac{n}{\ln n}, \quad b_n = \left(\frac{n + 1}{n}\right)^n \]

A. Both are convergent  
B. Both are divergent  
C. The first one is convergent while the second one is divergent  
D. The first one is divergent while the second one is convergent  
E. None of the above
3. Find all $p$ such that the series $\sum_{k=1}^{\infty} \sqrt{\frac{k^3 + 2k^2}{k^p + 1}}$ converges.

A. $p > 1$
B. $p > 6$
C. $p > 5$
D. $p \geq 5$
E. $p > 7$

4. $\sum_{k=0}^{\infty} \frac{2 + 2^k}{4^k} =$

A. $\infty$
B. 3
C. $\frac{8}{3}$
D. $\frac{14}{33}$
E. $\frac{14}{3}$

3
5. Find the sum of the telescoping series \( \sum_{n=2}^{\infty} \frac{2}{n^2 - n} \)

A. \( \frac{3}{2} \)
B. \( \infty \)
C. 3
D. 1
E. 2

6. The series \( \sum_{k=2}^{\infty} \frac{1}{k \ln k} \) is

A. divergent by comparison with \( \sum_{k=2}^{\infty} \frac{1}{k} \)
B. convergent by comparison with \( \sum_{k=3}^{\infty} \frac{1}{k^2} \)
C. divergent by integral test
D. convergent by integral test
E. a convergent \( p \) series.
7. Consider the alternating series
\[ \sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k} \]
Which of the following statements is/are correct?

I. The series converges absolutely

II. The series converges because \( \lim_{k \to \infty} \frac{1}{k} = 0 \) and \( \frac{1}{k} \) is always decreasing

III. The series diverges because \( \int_1^\infty |a_k(x)| \, dx = \int_1^\infty \frac{1}{x} \, dx \) diverges

A. I only
B. II only
C. I and II only
D. III only
E. All of them

8. What is the minimum number of terms to add in \( \sum_{k=8}^{\infty} \frac{(-1)^k}{k+1} \) to ensure that the partial sum is no more than 0.1 away from the sum of the infinity series?

A. 8
B. 9
C. 10
D. 11
E. 12
9. How many of the following series are conditionally convergent?

\[ \sum_{k=2}^{\infty} \frac{(-1)^k}{k \ln k} \]
\[ \sum_{k=1}^{\infty} \frac{(-1)^k}{\pi^k} \]
\[ \sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k^{0.998}} \]
\[ \sum_{k=1337}^{\infty} \frac{-1}{k^2} \]
\[ \sum_{k=1}^{\infty} \frac{(-1)^k}{k^\pi} \]

A. 1
B. 2
C. 3
D. 4
E. 5

10. On which of the following intervals does \( \sum_{k=1}^{\infty} \frac{2^k(x - 1)^k}{k!} \) converge absolutely?

A. \([-1, 1]\) only
B. \([-1, 1)\) only
C. \((-1, 1]\) only
D. \((-\infty, \infty)\)
E. This series never converges absolutely
11. The fourth-order Taylor polynomial of \( f(x) \) centered at \( x = \frac{5\pi}{6} \) is

\[
P_4(x) = \frac{1}{2} - \frac{\sqrt{3}}{2} \left( x - \frac{5\pi}{6} \right) - \frac{1}{4} \left( x - \frac{5\pi}{6} \right)^2 + \frac{\sqrt{3}}{12} \left( x - \frac{5\pi}{6} \right)^3 + \frac{1}{48} \left( x - \frac{5\pi}{6} \right)^4
\]

What is \( f''\left(\frac{5\pi}{6}\right) \)?

A. \( \frac{\sqrt{3}}{2} \)
B. \( \frac{1}{4} \)
C. \( \frac{1}{12} \)
D. \( \frac{1}{2} \)
E. \( \sqrt{3} \)

12. If a second-order Taylor series of \( f(x) = e^{-x} \) centered at \( x = a = 0 \) is used to estimate \( e^{-0.1} \), what is the maximum absolute error according to Taylor’s Estimation Theorem?

A. \( \frac{1}{5000} \)
B. \( \frac{1}{2000} \)
C. \( \frac{1}{800} \)
D. \( \frac{1}{600} \)
E. \( \frac{1}{1000} \)