MA 16200: Third Midterm Examiniation
Fall 2025, Purdue University

Exam version: 01

Name: _____   PUID #: _____

Exam Instructions:

- Follow these instructions carefully. Failure to do so may result in your exam being invalidated and/or an academic integrity violation. All suspected violations of academic integrity will be reported to the Office of the Dean of Students.

- Mark the circle of your recitation section below. Write your name and PUID on the top of this cover page **AND** the back cover page. **DO NOT WRITE ANYTHING ELSE** on either cover pages.

	Sec	Time	TA Name		Sec	Time	TA Name
○	121	7:30AM	Jonah Boan	○	910	12:30PM	Shivang Patel
○	109	7:30AM	Juliet Raginsky	○	117	12:30PM	Alex Yang
○	202	7:30AM	Dhruv Wadhwa	○	214	1:30PM	Fawzan Ali
○	123	8:30AM	Jonah Boan	○	208	1:30PM	Oliver Tan
○	111	8:30AM	Juliet Raginsky	○	119	1:30PM	Alex Yang
○	204	8:30AM	Dhruv Wadhwa	○	216	2:30PM	Fawzan Ali
○	902	9:30AM	Luke Miga	○	206	2:30PM	Oliver Tan
○	212	9:30AM	Kathryn Moran	○	224	3:30PM	Ayo Aitokhuehi
○	904	10:30AM	Luke Miga	○	103	3:30PM	Trey Crouch
○	210	10:30AM	Kathryn Moran	○	107	3:30PM	Marissa Munoz
○	115	11:30AM	Skip Moses	○	906	3:30PM	Andy Yu
○	218	11:30AM	Uyen Nguyen	○	222	4:30PM	Ayo Aitokhuehi
○	909	11:30AM	Shivang Patel	○	101	4:30PM	Trey Crouch
○	113	12:30PM	Skip Moses	○	105	4:30PM	Marissa Munoz
○	220	12:30PM	Uyen Nguyen	○	908	4:30PM	Andy Yu

- This exam consists of 12 questions for a total of 100 points.

- You have exactly one hour to complete the exam.

- Do not open the exam booklet or start writing before the proctor signals the start of the exam.

- Additional pages for scratch work can be found at the end of the booklet.

- Calculators, electronic devices, books, or notes are **NOT ALLOWED**.

- Students may not look at anybody else's exam, and may not communicate with anybody else except with their TA or instructor if there is a question.

- If you finish the exam before 7:25 pm, you may leave the room after turning in the exam booklet. You may not leave the room before 6:50 pm. If you don't finish before 7:25 pm, **YOU MUST REMAIN SEATED** until your TA comes and collects your exam booklet. You must stop working when the proctor signals the end of exam.

Good luck!

DO NOT DETACH THIS PAGE FROM THE EXAM BOOKLET.

Answer Sheet Instructions:

- Fill the circles completely with a **#2 PENCIL** for your answer choices. If you need to change your answer choice, erase the mark completely.

DO: ○ ●
DON'T: ⊘ ⊗ ⊘ ⊘ ⊙ ●

- Mark **EXACTLY ONE** circle for each question. Questions with more than one marked circle will receive no credit.

- Only what you marked on this page will be graded for score.

- Partial credit will not be awarded, unless otherwise indicated by individual questions.

Mark all your answer choices below:

	A	B	C	D	E	F	G	H	I	J
#1	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ	Ⓕ	Ⓖ	Ⓗ	Ⓘ	Ⓙ
#2	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ	Ⓕ	Ⓖ	Ⓗ	Ⓘ	Ⓙ
#3	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ	Ⓕ	Ⓖ	Ⓗ	Ⓘ	Ⓙ
#4	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ	Ⓕ	Ⓖ	Ⓗ	Ⓘ	Ⓙ
#5	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ	Ⓕ	Ⓖ	Ⓗ	Ⓘ	Ⓙ
#6	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ	Ⓕ	Ⓖ	Ⓗ	Ⓘ	Ⓙ

	A	B	C	D	E	F	G	H	I	J
#7	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ	Ⓕ	Ⓖ	Ⓗ	Ⓘ	Ⓙ
#8	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ	Ⓕ	Ⓖ	Ⓗ	Ⓘ	Ⓙ
#9	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ	Ⓕ	Ⓖ	Ⓗ	Ⓘ	Ⓙ
#10	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ	Ⓕ	Ⓖ	Ⓗ	Ⓘ	Ⓙ
#11	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ	Ⓕ	Ⓖ	Ⓗ	Ⓘ	Ⓙ
#12	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ	Ⓕ	Ⓖ	Ⓗ	Ⓘ	Ⓙ

Exam booklet version: ● 01   ○ 02   ○ Alternate   ○ Enlarged Text

Make no stray marks in the boxed region above. Do not modify the exam booklet version marking.

Remember to mark all your answer choices on the second cover page!

1. (8 points) Which of the following conclusion is correct if the ratio test is applied to the series

$$\sum_{n=1}^{\infty} \frac{3^n \cdot n^3}{n!} \quad ?$$

(A) The series is conditionally convergent because $r = 1$.

(B) The series is absolutely convergent because $r = 1/3$.

(C) The series is divergent because $r = 3$.

(D) The series is absolutely convergent because $r = 0$.

(E) The ratio test is inconclusive because $r = 1$.

2. (8 points) The sequence $\{a_n\}_{n=1}^{\infty}$ defined by the explicit formula

$$a_n = \frac{n}{n^2 + 1}$$

has a limit $L = 0$. What are all indices n that satisfy $|a_n - L| < 0.1$?

(A) $n \geq 5$

(B) $n \geq 10$

(C) $n \geq 15$

(D) $n \geq 20$

(E) All positive integer n

3. (9 points) Given two arbitrary series $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$, determine which of the following three statements is/are always TRUE.

    (i) If $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ are both convergent, then $\sum_{n=1}^{\infty} a_n - b_n$ is convergent.

    (ii) If $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ are both divergent, then $\sum_{n=1}^{\infty} a_n + b_n$ is divergent.

    (iii) If $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ are both divergent, then $\sum_{n=1}^{\infty} a_n b_n$ is divergent.

Note: Partial credit is possible for this question.

(A) None of the above

(B) Only (i)

(C) Only (ii)

(D) Only (iii)

(E) Only (i) and (ii)

(F) Only (i) and (iii)

(G) Only (ii) and (iii)

(H) All of (i), (ii), and (iii)

4. (10 points) Determine whether each of the following series is absolutely convergent, conditionally convergent, or divergent.

$$S_1 = \sum_{n=1}^{\infty} \frac{(-3)^n}{2^{3n}}, \quad \text{and} \quad S_2 = \sum_{n=1}^{\infty} \frac{n^2 \cos(n\pi)}{n^3 + 1}$$

(A) S_1 is absolutely convergent; S_2 is absolutely convergent.

(B) S_1 is absolutely convergent; S_2 is conditionally convergent.

(C) S_1 is absolutely convergent; S_2 is divergent.

(D) S_1 is conditionally convergent; S_2 is absolutely convergent.

(E) S_1 is conditionally convergent; S_2 is conditionally convergent.

(F) S_1 is conditionally convergent; S_2 is divergent.

(G) S_1 is divergent; S_2 is absolutely convergent.

(H) S_1 is divergent; S_2 is conditionally convergent.

(I) S_1 is divergent; S_2 is divergent.

Note: Partial credit is possible for this question.

5. (8 points) What is the result of using the third-order Taylor polynomial for $f(x) = \cos(x)$ centered at the origin to approximate the value of $\cos(1)$?

(A) $1/2$

(B) $3/2$

(C) $13/24$

(D) $11/24$

(E) $37/24$

6. (8 points) A sequence $\{z_n\}_{n=1}^{\infty}$ is defined by the recurrence relation

$$z_1 = -1, \quad \text{and} \quad z_{n+1} = (z_n)^2 - 3.$$

What is z_{2025}?

(A) 2

(B) -2

(C) 2022

(D) 1

(E) -1

7. (8 points) If we use the second-order Taylor polynomial $p_2(x)$ for $f(x) = \sqrt{x}$ centered at $a = 4$ to approximate the value of $\sqrt{4.1}$, which one of the following statements is a correct application of Taylor's remainder theorem about the remainder $R_2(4.1)$?

The derivatives of $f(x)$ are listed below:

$$f'(x) = \frac{1}{2}x^{-1/2}, \quad f''(x) = -\frac{1}{4}x^{-3/2}, \quad \text{and} \quad f'''(x) = \frac{3}{8}x^{-5/2}.$$

(A) $|R_2(4.1)| = \dfrac{|f''(c)|}{2!} \cdot |4.1|^2 \leq \dfrac{|f''(4.1)|}{2!} \cdot (4.1)^2$ for some $4 \leq c \leq 4.1$.

(B) $|R_2(4.1)| = \dfrac{|f''(c)|}{2!} \cdot |0.1|^2 \leq \dfrac{|f''(4)|}{2!} \cdot (0.1)^2$ for some $4 \leq c \leq 4.1$.

(C) $|R_2(4.1)| = \dfrac{|f'''(c)|}{3!} \cdot |0.1|^3 \leq \dfrac{|f'''(4.1)|}{3!} \cdot (0.1)^3$ for some $4 \leq c \leq 4.1$.

(D) $|R_2(4.1)| = \dfrac{|f'''(c)|}{3!} \cdot |4.1|^3 \leq \dfrac{|f'''(4)|}{3!} \cdot (4.1)^3$ for some $4 \leq c \leq 4.1$.

(E) $|R_2(4.1)| = \dfrac{|f'''(c)|}{3!} \cdot |0.1|^3 \leq \dfrac{|f'''(4)|}{3!} \cdot (0.1)^3$ for some $4 \leq c \leq 4.1$.

8. (8 points) Evaluate the limit of the sequence

$$\left\{ \frac{\ln(n+3)}{\ln(4n+1)} \right\}_{n=1}^{\infty}$$

(A) $\ln(4)$

(B) 1

(C) 4

(D) $1/4$

(E) The sequence is divergent.

9. (8 points) What are all values of p that can make the following series convergent?

$$\sum_{n=1}^{\infty} \frac{(\arctan(n))^p}{n^2+1}$$

(A) $p < -1$

(B) $p \le -1$

(C) $p > 1$

(D) p can be any real number.

(E) There is no value of p that makes the series convergent.

10. (9 points) Which of the following convergence tests is/are applied correctly to determine that the following series is convergent?

$$\sum_{n=1}^{\infty} \frac{e^n}{e^{2n} - 1}$$

   (i) The series is convergent by the integral test.

   (ii) The series is convergent by the ratio test.

   (iii) The series is convergent by the direct comparison test with $\sum \frac{1}{e^n}$.

Note: Partial credit is possible for this question.

(A) None of the above

(B) Only (i)

(C) Only (ii)

(D) Only (iii)

(E) Only (i) and (ii)

(F) Only (i) and (iii)

(G) Only (ii) and (iii)

(H) All of (i), (ii), and (iii)

11. (8 points) Evaluate the sum of the series

$$\sum_{n=0}^{\infty} \frac{4 + (-2)^n}{3^n}$$

(A) 9

(B) $3/2$

(C) $33/5$

(D) $3/5$

(E) The series is divergent.

12. (8 points) Consider the series

$$S = \sum_{k=1}^{\infty} \frac{(-1)^k}{(4k+3)^2}.$$

By the error estimate for alternating series, how many terms are needed for the partial sum S_n to get within 10^{-4} of the true sum S?

(A) 26

(B) 25

(C) 2499

(D) 24

(E) 2500

Remember to mark all your answer choices on the second cover page!
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