1. Which of the following series converges?

(I) $\displaystyle\sum_{n=1}^{\infty} \frac{2}{n^{0.99}}$

(II) $\displaystyle\sum_{n=1}^{\infty} \frac{1-2\sqrt{n}}{n^2}$

(III) $\displaystyle\sum_{n=1}^{\infty} \frac{1-2\sqrt{n}}{n^{3/2}}$

A. All of them.

B. (II) only.

C. (I) and (II) only.

D. (II) and (III) only.

E. (I) only.

2. Which of the following statements is true?

(I) If $0 \le a_n \le b_n$ and $\sum b_n$ converges, then $\sum a_n$ converges.

(II) If $a_n \ge b_n \ge 0$ and $\sum b_n$ diverges, then $\sum a_n$ diverges.

(III) If $0 \le a_n \le b_n$ and $\sum b_n$ diverges, then $\sum a_n$ diverges.

A. (I) only.

B. (II) only.

C. (I) and (III) only.

D. (II) and (III) only.

E. (I) and (II) only.

3. Which of the following series converges?

(I) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{\sqrt{n^3+1}}$

(II) $\displaystyle\sum_{n=1}^{\infty} \frac{n+3^n}{n+5^n}$

(III) $\displaystyle\sum_{n=1}^{\infty} \frac{n+3}{(n+2)^3}$

A. All of them.

B. (I) and (II) only.

C. (I) and (III) only.

D. (II) and (III) only.

E. (I) only.

4. Which of the following statements is correct (only one of them is correct):

A. The series $\displaystyle\sum_{n=1}^{\infty} \sin\frac{0.1}{n^2}$ converges, by the limit comparison test.

B. The series $\displaystyle\sum_{n=1}^{\infty} \sin\frac{0.1}{n^2}$ converges, because $\sin\frac{0.1}{n^2} \to 0$, as $n \to \infty$.

C. The series $\displaystyle\sum_{n=1}^{\infty} \sin\frac{0.1}{n^2}$ is an alternating series, and therefore is convergent.

D. The series $\displaystyle\sum_{n=1}^{\infty} \sin\frac{0.1}{n^2}$ diverges by the ratio test.

E. $\displaystyle\sum_{n=1}^{\infty} \sin\frac{0.1}{n^2}$ diverges, because the integral $\displaystyle\int_1^{\infty} \sin x\,dx$ is divergent.

5. For the series

(I) $\displaystyle\sum_{n=2}^{\infty} \frac{(-1)^n}{\ln n}$

(II) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{e^{\frac{1}{n}}}{n^3}$

A. (I) and (II) are absolutely convergent.

B. (I) is divergent, (II) is absolutely convergent.

C. (I) is conditionally convergent, (II) is absolutely convergent.

D. (I) and (II)are conditionally convergent.

E. (I) is divergent, (II) is conditionally convergent.

6. The series $\displaystyle\sum_{n=1}^{\infty} \frac{(-2)^n}{n^n}$

A. Converges absolutely by comparison with $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^n}$.

B. Diverges since $\displaystyle\lim_{n\to\infty} \frac{(-2)^n}{n^n} \neq 0$.

C. Converges absolutely by the root test.

D. Diverges by the ratio test.

E. Diverges by the root test.

7. Consider the following series:

I. $\sum_{n=1}^{\infty} (-1)^n \frac{n}{n+2}$

II. $\sum_{n=1}^{\infty} \frac{1}{n+3^n}$

III. $\sum_{n=1}^{\infty} (-1)^n \frac{n}{n^2+2}$

A. They all converge.

B. Only (I) and (II) converge.

C. Only (I) and (III) converge.

D. Only (II) and (III) converge.

E. They all diverge.

8. Consider the following series:

I. $\sum_{n=1}^{\infty} \frac{n^2}{n+1} \frac{1}{2^n}$

II. $\sum_{n=1}^{\infty} (-1)^n \frac{n+1}{n^2}$

III. $\sum_{n=1}^{\infty} \frac{n+1}{n^2}$

A. They all converge.

B. Only (I) and (II) converge.

C. Only (I) and (III) converge.

D. Only (II) and (III) converge.

E. They all diverge.

9. Find the radius of convergence of $\sum_{n=0}^{\infty} \sqrt{n}2^n x^n$.

A. 0

B. $\frac{1}{2}$

C. 1

D. 2

E. ∞

10. Given that the radius of convergence of $\sum_{n=0}^{\infty}(-1)^n \frac{(x-3)^n}{2n+1}$ is 1, find the interval of convergence.

A. $(2, 4)$

B. $[2, 4]$

C. $[2, 4)$

D. $(2, 4]$

E. None of the above.

11. Find a power series for the indefinite integral $F(t) = \int \frac{t}{1-t^8}\,dt$ and find its radius of convergence R.

A. $F(t) = C + \sum_{n=0}^{\infty} \frac{t^{8n+2}}{8n+2}, \; R = 1$

B. $F(t) = C + \sum_{n=0}^{\infty} \frac{t^{8n+2}}{8n+2}, \; R = \infty$

C. $F(t) = C + \sum_{n=0}^{\infty} t^{8n+1}, \; R = 1$

D. $F(t) = C + \sum_{n=0}^{\infty} t^{8n+1}, \; R = \infty$

E. None of the above.

12. Find the Taylor series for $f(x) = e^{2x}$ centered at $a = 3$.

A. $\sum_{n=0}^{\infty} \frac{2^n x^n}{n!}$

B. $\sum_{n=0}^{\infty} \frac{2^n}{n!} e^3 (x+3)^n$

C. $\sum_{n=0}^{\infty} \frac{2^n}{n!} e^6 (x+3)^n$

D. $\sum_{n=0}^{\infty} \frac{2^n}{n!} e^3 (x-3)^n$

E. $\sum_{n=0}^{\infty} \frac{2^n}{n!} e^6 (x-3)^n$